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FINAL TECHNICAL REPORT

FRACTURE AND FATIGUE OF BI-MATERIALS

F44620-74-C-0023

Period 1 December 1973 to 30 September 1978

MIT OSP 81497

Reported by James W. Mar
Professor of Aeronautics & Astronautics

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INTRODUCTION

This final technical report describes the research performed under USAF Office of Scientific Research Contract F 44620-74-C-0023 entitled "Fracture and Fatigue of Bi-Materials" during the period 1 December 1973 to 30 September 1978. The AFOSR technical monitors were Mr. William J. Walker and Lt. Col. Joseph D. Morgan III.

As suggested by the title of this research contract, the research is directed toward a fundamental understanding of the failure mechanisms of the filamentary composites such as boron/aluminum and graphite/epoxy which are technologically the most important examples of bi-materials. At the beginning of the program there was an attempt to use the diffusion bonding process to create a special material which could serve as a model for bi-material behavior but the diffusion bonding process proved to be much too expensive and the attention was thereafter focussed on the available filamentary composite materials.

The penultimate objectives of this research are the theoretical foundations and the experimental justifications for a structural design methodology to be used for the design of filamentary composite materials to meet USAF damage tolerance and durability requirements. To that end the work has both theoretical and experimental phases.

Analytically, the structural combination of filament and matrix can be viewed from two different geometric scales. At what is termed the "macroscopic" scale, it is assumed the filamentary composite is orthotropic and homogeneous. The other scale is termed "micro" and views the filamentary composite as a bi-material.

On a macroscopic scale, the stress analysis of cracks in orthotropic materials and on a microscopic scale, the stress singularity and stress distributions of a crack lying at the interface of a bi-material have been studied.

The experimental work has encompassed the compression-compression fatigue of graphite/epoxy laminates with holes, the tensile fracture of uni-directional laminates with circular discontinuities, and the tensile fracture of cross-plied laminates with various geometric configurations of discontinuities.

SUMMARY OF MAJOR FINDINGS

1. As a result of our experimental investigation of boron/aluminum bi-materials[5] we have proposed a formula to predict the rapid fracture of $[\pm 45/0]_s$ boron/aluminum laminates. The motivation for this theory comes from linear elastic fracture mechanics as applied to homogenous materials where the fracture stress, σ_f, in the presence of a crack in a wide sheet is given by

$$\sigma_f = \sqrt{\pi}\, K_{IC}[2a_c]^{-1/2} \tag{1}$$

Thus, K_{IC} is a material parameter called the fracture toughness which is experimentally determined and which has dimensions of stress x length to the one-half power. The crack size effect (2a is the length of the crack) is embodied in the exponent of "minus one-half" which is the order of the mathematical singularity at the tip of the crack.

The present theory proposes an equation for the fracture stress, σ_f, of filamentary composites of the form

$$\sigma_f = H_c(2a_c)^{-m} \tag{2}$$

where H_c is akin to K_{IC} but which has dimensions of "stress x length to the m power" (which is different than that of K_{IC}). The exponent "minus m" is the order of the singularity of a crack lying in the matrix with its tip at the interface of matrix and filament. It has been shown that the order of the singularity is a function of the ratio μ_1/μ_2 of the shear moduli of the matrix and filament and of the two Possion ratios ν_1 and ν_2.

Not much statistical data on the shear moduli of boron filaments, graphite filaments and the epoxy resins are available. If the shear moduli of the boron and 6061 aluminum are taken as 24 million psi and 3.8 million psi respectively, then the ratio of moduli for the boron/aluminum system is approximately .16. If the graphite filament and the epoxy resin tensile moduli are taken as 28 and 17 million psi, respectively, then the ratio is approximately .025. The most likely value for m in a boron/aluminum system is about .33 and for a graphite/epoxy system is .26.

The best fit to our experimental data yields the result

$$\frac{\sigma_f}{\sigma_0} = .33(2a_c)^{-.30} \tag{3}$$

where σ_0 is the strength of the laminate without holes or other forms of discontinuities.

Imbued with this success we applied this formula to experimental data gathered by other experimenters on graphite/epoxy laminates.[4] This has yielded the relation

$$\frac{\sigma_f}{\sigma_0} = .43(2a_c)^{-.23} \tag{4}$$

for $[0/\pm45/90]_s$ symmetric layups and

$$\frac{\sigma_f}{\sigma_0} = .33(2a_c)^{-.25} \tag{5}$$

for $[0/\pm45]_s$ symmetric layups.

Some observations and comments on these results may be helpful:

a. As predicted by the theory, the exponent in the boron/aluminum material is larger than for the graphite/epoxy material.

b. The experimental values are smaller than predicted by theory.

c. It should be possible to correlate the factor H in Eq. (3) to the stacking sequence of the layup. At any hole size, 2a, the ratio of the respective σ_f/σ_0 values for the two graphite/epoxy layups is about .77 (i.e., .33 ÷ .43). The ratio of 0° plies in the two graphite/epoxy layups is .75, i.e. one-third of the plies in the $[0/\pm45]$ layup is oriented at 0° while one-fourth is the proportion for the $[0/\pm45/90]$ layup. Obviously, much more experimental work is required before such a simple correlation can be used.

d. The filaments in the ±45 plies in a layup contribute to the "toughness" whereas the ones at 90° do not because fracture can occur solely in the matrix of the 90° plies.

e. Implicit in the proposed correlation is the assumption that cracks in the matrix perpendicular to the 0° filaments are the triggering mechanism for the final failure. Cracks in the matrix at an angle to the 45° filaments have a different kind of a singularity and hence induce an intensity of stress in these filaments insufficient to cause catastrophic failure in the layup. However, the exponent m may be a function of the singularities presented by the cracks at the non-zero degree filaments.

2. Uni-directional filamentary/matrix of bi-materials were tested in the presence of circular hole and slit types of discontinuities.[9] The failure process of such a uni-directional laminate begins as a 'slit', i.e., a crack propagating in the matrix parallel to the filaments. With the aid of crack propagation gages we were able to correlate the stress at which the slit initiates against the length of the discontinuity. This correlation leads us to suggest that linear elastic fracture mechanics may be used and that the conset of splitting is governed by an equation of the form

$$\sigma_s = H_s (2a)^{-.50} \tag{6}$$

where σ_s is the far field stress at the onset of splitting, (2a) is the length of the discontinuity and H_s is a fracture toughness against splitting.

3. We have tested a small number of $[0/\pm 45]_s$ and $[0/\pm 30]_s$ graphite/epoxy laminates in the presence of circular hole discontinuities. There is a

a difference in the failure modes of these two laminates. Both sets of data do correlate with the theory proposed in Section I and preliminary analyses show that the substance of comment (le) needs additional investigation.

4. Static compression tests of $[\pm 45/0]_s$ and $[0/\pm 45]_s$ laminates[6] were carried out prior to compression-compression fatigue tests. These laminates, which were the facings of sandwich beams, were of three different configurations: (i) unflawed, (ii) a .25 inch diameter hole, and (iii) a .25 inch diameter hole clamped by washers held together by a loose fitting bolt. The results are shown in Fig. 1. Some observations can be made.

   (a) The mean strength of the $[\pm 45/0]_s$ specimens is higher than for the $[0/\pm 45]_s$ specimens. This agrees with the notion that the free edge interlaminar stresses normal to the plane of the laminate are compression for the $[\pm 45/0]_s$ system and tension for the $[0/\pm 45]_s$ system. The interlaminar tensions at the free edge degrade the compression strength.

   (b) Once a hole is drilled, the mean strengths for the two systems are not appreciably different. This suggests that the strain concentrations caused by the holes dominate the free edge effect.

   (c) The clamped hole specimens exhibit about the same mean strengths.

   (d) The $[\pm 45/0]_s$ specimens appear to exhibit a larger scatter than do the $[0/\pm 45]_s$ specimens.

5. Schematic representations which compare the static and fatigue failure modes of the $[\pm 45/0]_s$ and $[0/\pm 45]_s$ laminates as observed in our fatigue

tests are shown in Figs. 2 and 3 respectively.[5] Observations are as follows:

(a) There are significant differences in the failure modes between the two laminate systems.

(b) The unflawed $[0/\pm 45]_s$ specimens exhibit the same kinds of failure in both static and fatigue failures. This is not true for the $[\pm 45/0]_s$ system.

(c) Generally, the fatigue modes of failure are different than the static modes [with the exceptions noted in (a)]. This points up the added complexities involved in damage tolerance and durability analyses. In metals the final failures, whether due to cyclic or static applications of the loads, are the same basic phenomenon and hence can be predicted by the same fracture mechanics formula. This fortuitous situation does not apply to the behavior of the advanced composites. The varieties of failure modes and the differences in failure modes between static and fatigue in the advanced composites points up the need for continued research.

(d) There does appear to be some beneficial effect of the clamping action provided by the washers, as can be seen in Fig. 4. More data is required but if the trends exhibited in Fig. 4 are verified, then perhaps the beneficial effect of the clamping provided by mechanical fasteners can be used for design.

6. A much larger number of compression-compression[7] fatigue tests on laminates containing .25 inch diameter circular holes were carried out on the laminate systems denoted by $[0/\pm30]_s$, $[\pm30/0]_s$, $[0/\pm45]_s$ and $[\pm45/0]_s$. There was a large difference in the minimum and maximum number of cycles, N, to failure as shown in the accompanying table.

	No. of Specimens	Min. N	Max. N	Weibull Parameters α	Weibull Parameters β	Max. Stress Ksi
$[0/\pm30]_s$	12	1	34	1.4	9	49
$[\pm30/0]_s$	12	3	37	1.4	13	53
$[0/\pm45]_s$	11	6	121	1.6	50	49
$[\pm45/0]_s$	12	10	331	1.6	190	49

The maximum likelihood estimates for the parameters α and β in the Weibull equation

$$P(N) = 1 - e^{-\left(\frac{N}{\beta}\right)^{\alpha}} \qquad (7)$$

are also shown in the table and the plots for the Weibull distributions are shown in Fig. 5. There is a large variation in N as graphically illustrated in the figure.

7. Damage at the hole was visually detected very early in the cyclic testing at the holes in all of the specimens.[7] For example, there are 11 specimens in the $[0/\pm45]_s$ group. Of these one failed at 6000 cycles. Nine

of the remaining 10 showed visually discernible damage at 10,000 cycles. The remaining specimen which did not exhibit damage at 10,000 cycles exhibited damage at 20,000 cycles. The $[\pm 45/0]_s$ group of specimens exhibited similar behavior. For the other two groups, $[0/\pm 30]_s$ and $[\pm 30/0]_s$, most of the specimens exhibited damage at the hole by 3000 cycles. These results lead to the following comments:

a. Considerable damage tolerance was demonstrated by the two groups which had ± 45 plies in that the numbers of cycles from visual detection of damage at the hole to final failure were large.

b. It can be postulated that the initial damage at the hole immediately reduces the high strain concentration caused by the hole because the strain concentration exists only if the surface of the hole and the surrounding material remains continuous. Once the surface of the hole is damaged by splits or delaminations, then the "soundness" of the structure is breached and the continuous strain fields which lead to the high concentration cannot exist. Thus, unlike metals and in direct contradistinction to metals, the initiation of damage at a hole leads to a reduction of the strain concentration and may in part explain the damage tolerance of these filamentary systems. In metals, the initiation of cracks at a hole means a transition from a stress concentration factor approach to a fracture mechanics approach in order to account for the theoretically infinite concentration at the crack tip. This aspect of the behavior of composite materials, if exploited, would have great technological impact.

c. These results seem to indicate the range of cycles at which damage can be detected at the hole is appreciably less than the range of cycles which encompasses failure. This is different than for metals because crack initiation in metals requires a scatter factor of four but once a crack has initiated, its growth to fracture can be encompassed by a factor significantly less than four. Our experimental results suggest the opposite for the filamentary composites. This may be due in part to the fact that the observed damage mechanisms increases in complexity with the accumulation of loading cycles.

3. The theoretical and analytical studies on the fracture and fatigue of bi-materials has addressed the macroscopic stress analysis and the micro-mechanics of cracks (Appendix A). These studies occurred early in the program and provided the theoretical foudations for our fracture theory. Our experimental studies have given us a phinomenological insight and data base upon which these earlier analytical studies can be applied. The stress distribution and stress intensities of cracks in (1) homogeneous materials with complicated geometry (2) bi-material interface (3) anisotropic materials have been made amenable for simplified and economical solution by the creation of crack-containing finite elements. A family of bi-material, homogeneous, and anistropic special crack elements in plane and axisymmetric cases have been developed. These include

a. Homogeneous mixed-mode plane crack element

b. Bi-material debonding plane crack element

c. Bi-material perpendicular plane crack element

d. Axisymmetric penny-shaped crack element

e. Axisymmetric circumferential crack element

f. Axisymmetric bi-material debonding crack element

g. Anisotropic plane crack element

PUBLICATIONS & PERSONNEL RESEARCH EFFORT

1. Lin, K. Y., "Fracture of Filamentary Composite Materials," Ph.D. Dissertation, Dept. of Aero. & Astro., M.I.T., Cambridge, MA, Jan. 1977.

2. Lin, K. Y. and Mar, J. W., "Finite Element Analysis of Stress Intensity Factors for Cracks at a Bi-Material Interface," International Journal of Fracture, Vol. 12, Aug. 1976, pp. 521-531.

3. Lin, K. Y., Tong, P. and Orringer, O., "Effect of Shape and Size on Hybrid Crack-Containing Finite Elements," Presented at the Second ASME Pressure Vessels and Piping Conference, San Francisco, CA, June 23-27, 1975.

4. Mar, J. W. and Lin, K. Y., "Fracture Mechanics Correlation for Tensile Failure of Filamentary Composites with Holes," Journal of Aircraft, Vol. 14, No. 7, July 1977, pp. 703-704.

5. Mar, J. W. and Lin, K. Y., "Fracture of Boron/Aluminum Composites with Discontinuities," Journal of Composite Materials, Oct. 1977, pp. 405-421.

6. Maass, D. P., "The Effect of Compression Fatigue on Failure Modes of Graphite/Epoxy Laminates," S.M. Dissertation, Dept. of Aero. & Astro., M.I.T., Cambridge, MA, Sept. 1977.

7. Graves, M. J., "The Effects of Compression-Compression Fatigue on Balanced Graphite/Epoxy Laminates With Holes," S.M. Dissertation, Dept. of Aero. & Astro., M.I.T., Cambridge, MA, Feb. 1979.

8. Orringer, O., Lin, K. Y., Stalk, G., Tong, P., and Mar, J. W., "K-Solutions with Assumed-Stress Hybrid Elements," Journal of the Structural Division, ASCE, Vol. 3, No. ST2, Feb. 1977, pp. 321-334.

9. Mar, J. W. and Lin, K. Y., "Characterization of Splitting Process in Graphite/Epoxy Composites," Journal of Composite Materials, October 1979, pp. 278-287.

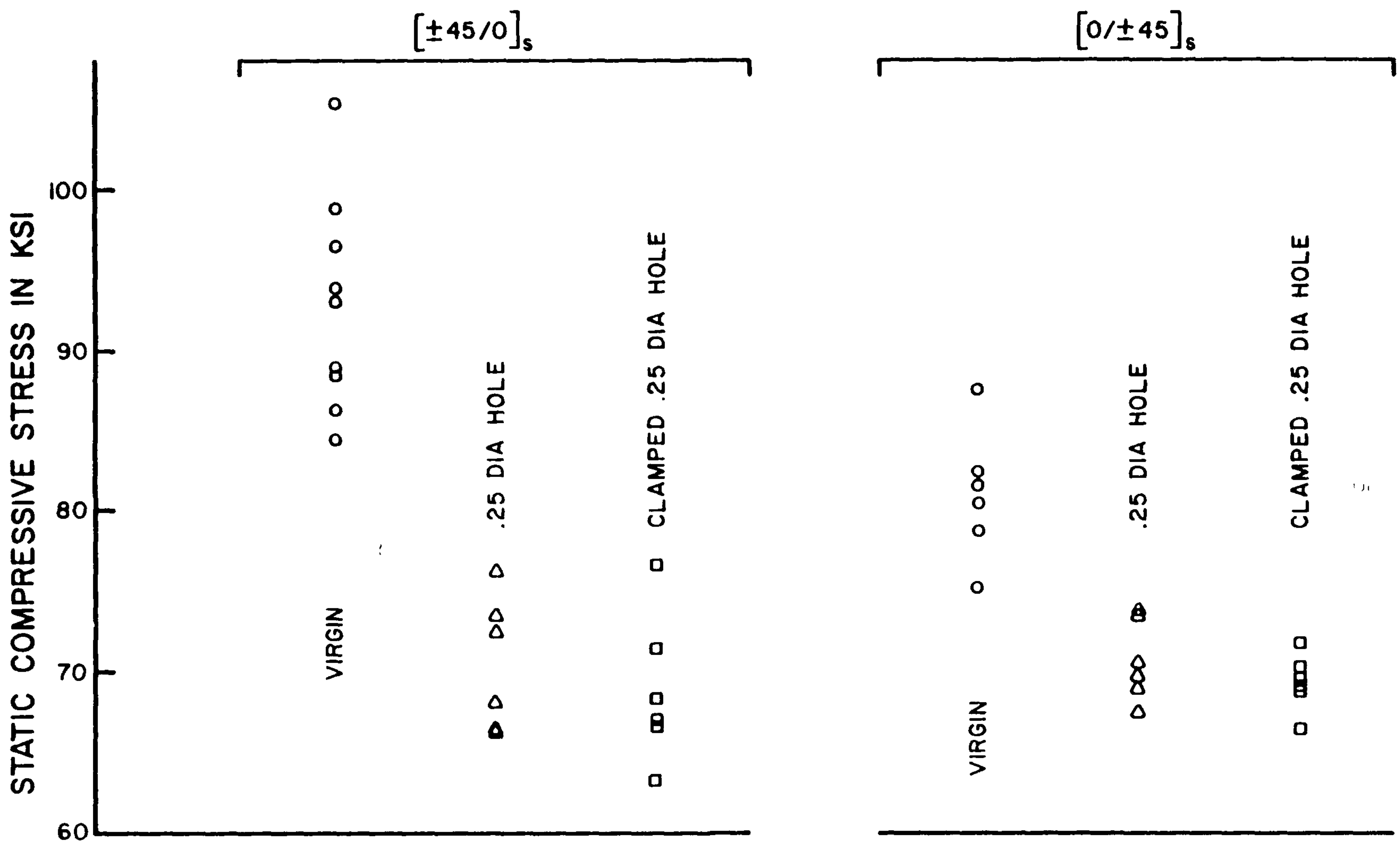


FIG. 1: STATIC COMPRESSIVE STRENGTH RESULTS

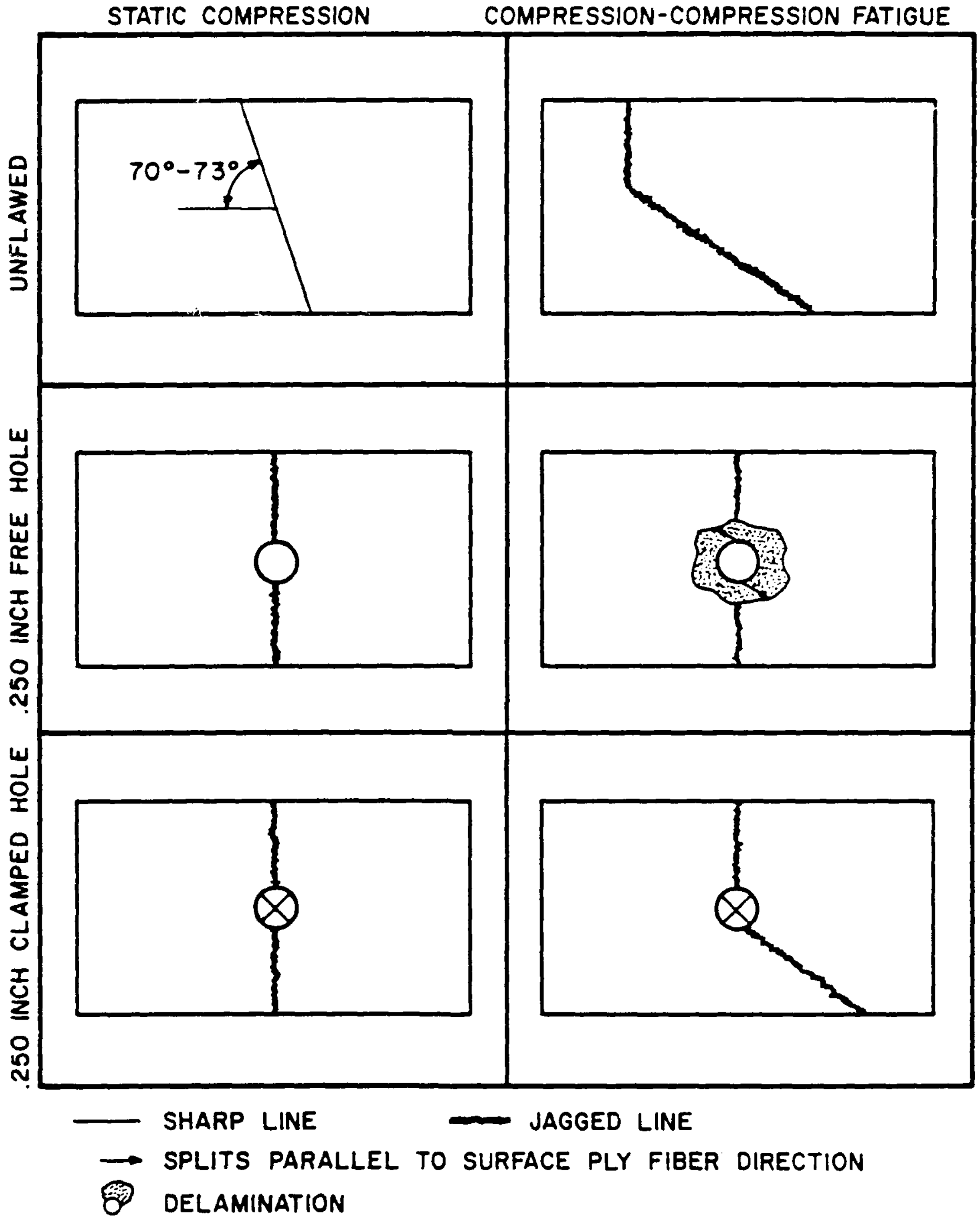
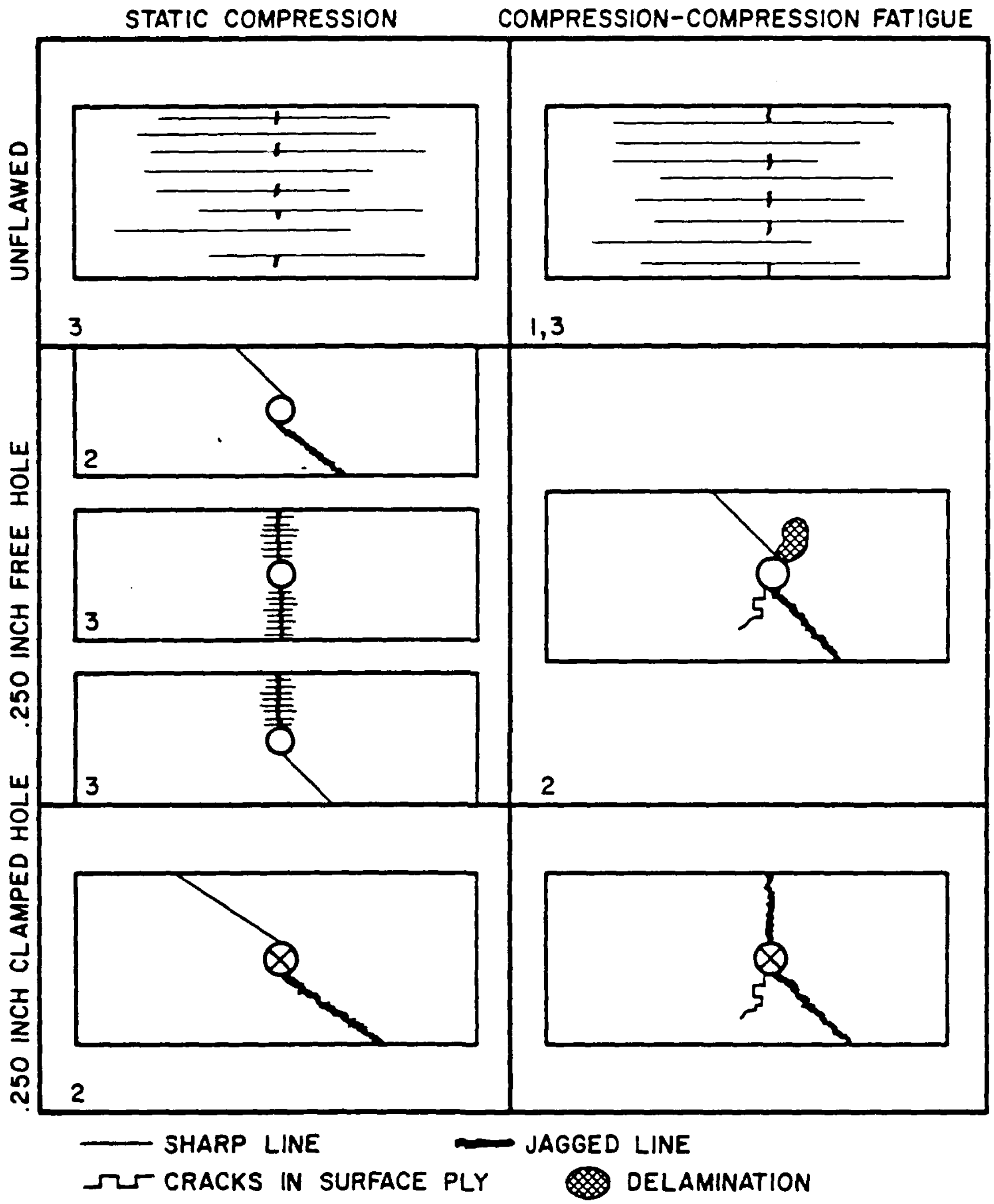


FIG. 2: SCHEMATIC REPRESENTATION OF FAILURE MODES - $[\pm 45/0]_s$ LAMINATE



NOTES: 1. SEQUENTIAL BUCKLING BEGINNING AT 12 25 % OF LIFE

2. SHARP AND ROUGH LINES OBSERVED IN SAME

3. (0) SURFACE PLY DETACHED AND BUCKLED

FIG. 3: SCHEMATIC REPRESENTATIONS OF FAILURE MODES - $[0/\pm45]_s$ LAMINATE

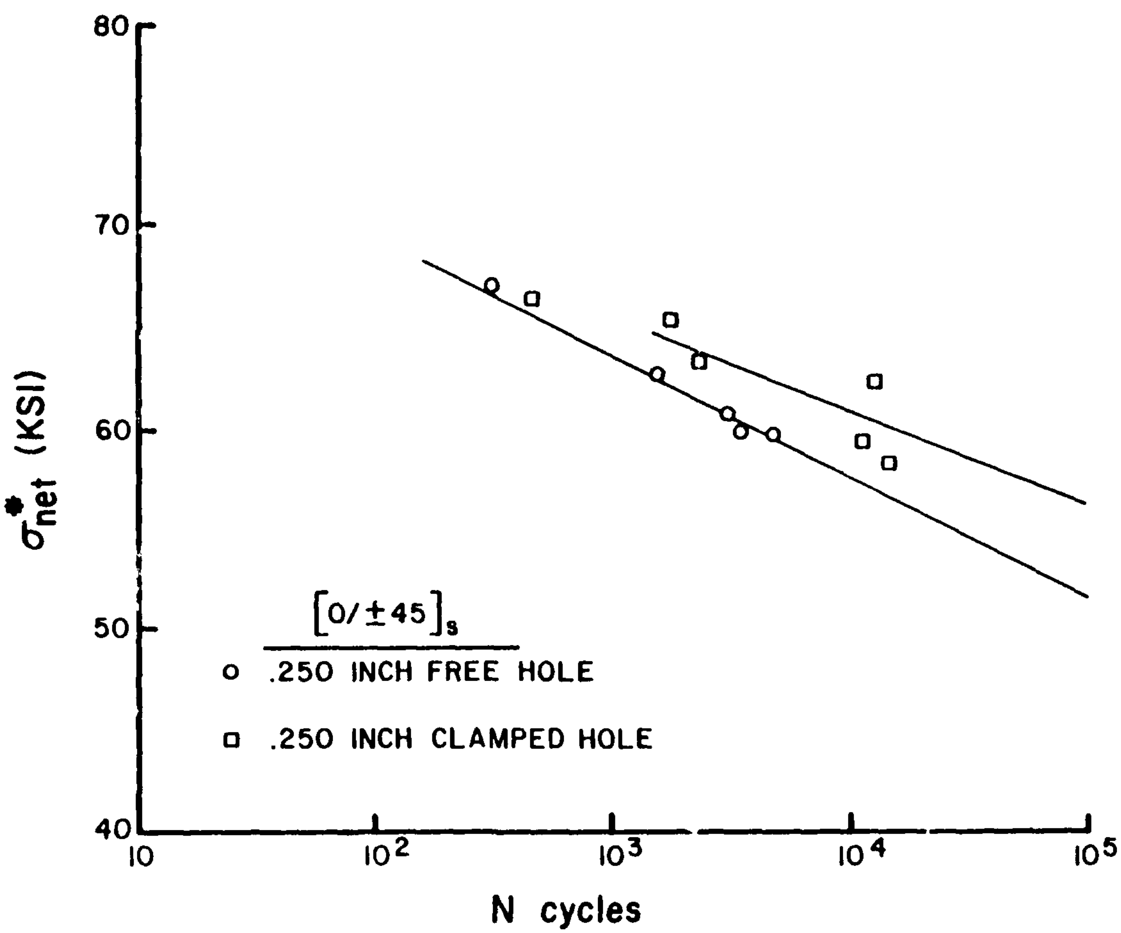
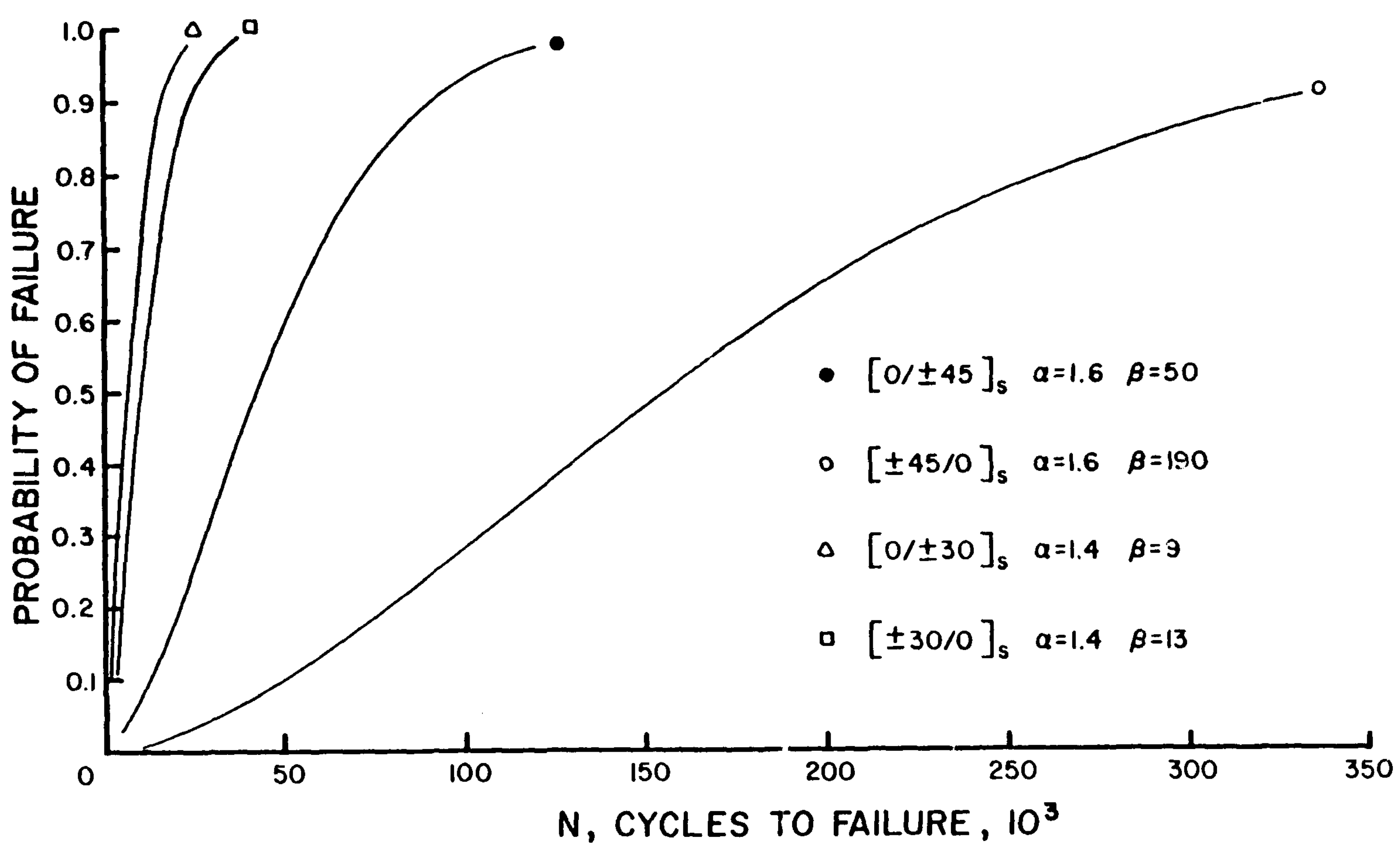


FIG. 4: LAMINATE: FREE HOLE VS. CLAMPED HOLE

FIG. 5: FATIGUE: WEIBULL PLOT - ALL DATA



LIST OF APPENDICES

APPENDIX A

FRACTURE MECHANICS OF COMPOSITES

Introduction

The concepts of fracture mechanics are now being applied to the study of crack extension phenomena, both slow and catastrophic, in isotropic homogeneous materials, and is of great importance wherever high strength metallic alloys are used. Based on energy equilibrium considerations, Griffith (1920) first developed a fracture theory for brittle materials such as glass [4]. Griffith's theory postulates that fracture occurs when the decrease of elastic strain energy per increment of crack area is equal to the increase of surface energy over the same area. However, the surface energy for a solid is difficult to calculate and measure. To overcome this difficulty, Irwin introduced the concept of "stress intensity factor" K, which is related to the amplitude of the stress field near the tip of a crack [5]. In fact, it can be shown that the stress intensity factor is related to the rate of energy release per unit area, $G \equiv \frac{\partial U}{\partial A}$, and hence to Griffith's surface energy. The Irwin-Griffith fracture criterion states that catastrophic crack propagation occurs when G or K reaches a critical value G_c or K_c and this is regarded as a material property, called "the fracture toughness" of the material.

It can be shown that under general loadings, the stress distribution of an elastic body in the presence of cracks can be superimposed from three separate cases: "mode 1 which is the opening mode", "mode 2 which is the shearing mode", and "mode 3 which is the out-of-plane shearing mode" [6]. For homogeneous isotropic materials, values and formulae of stress intensity factors obtained from various methods of analyses can be found in various handbooks [7]. A large body of laboratory data has also been generated and there is sufficient evidence that fracture toughness K_{I_c} is a material constant, independent of crack length and crack configuration.

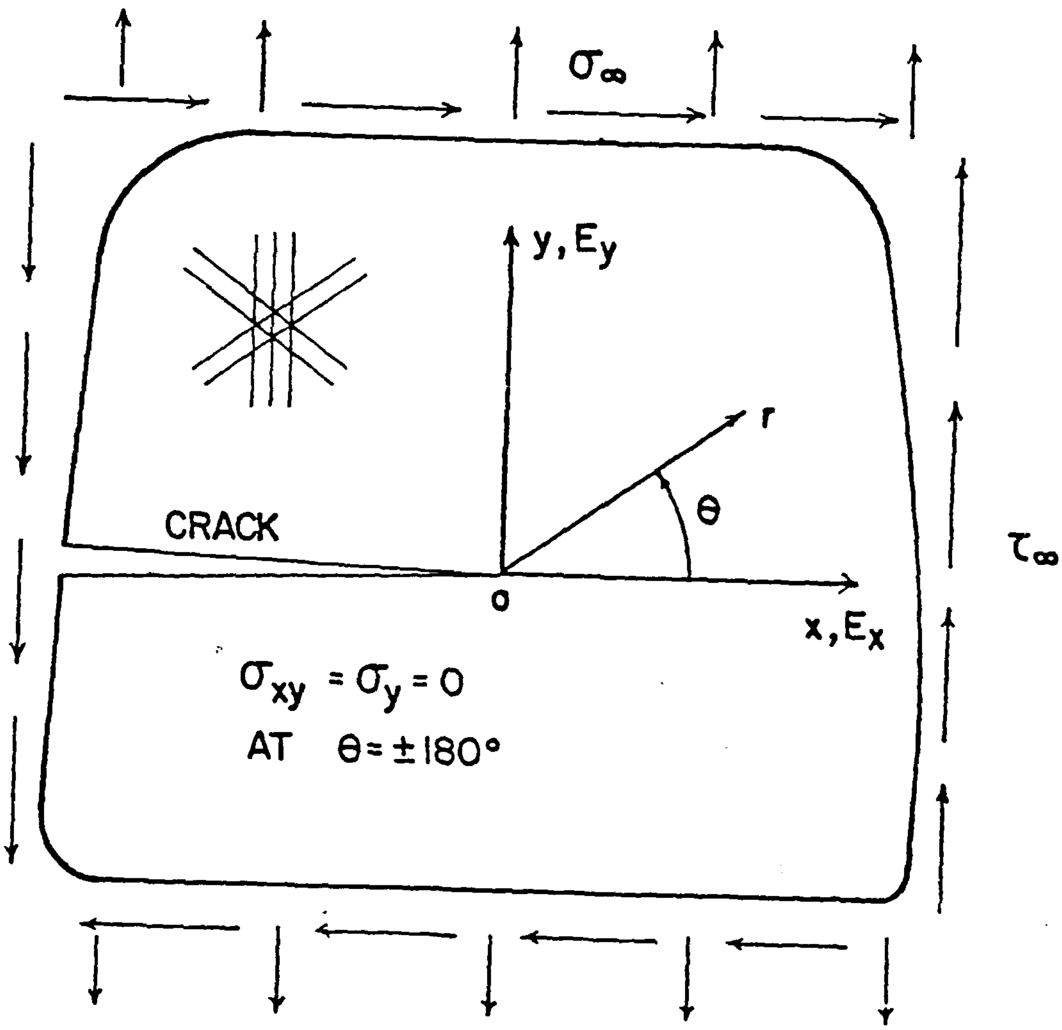


Figure 1. Crack in a Homogeneous Anisotropic Sheet

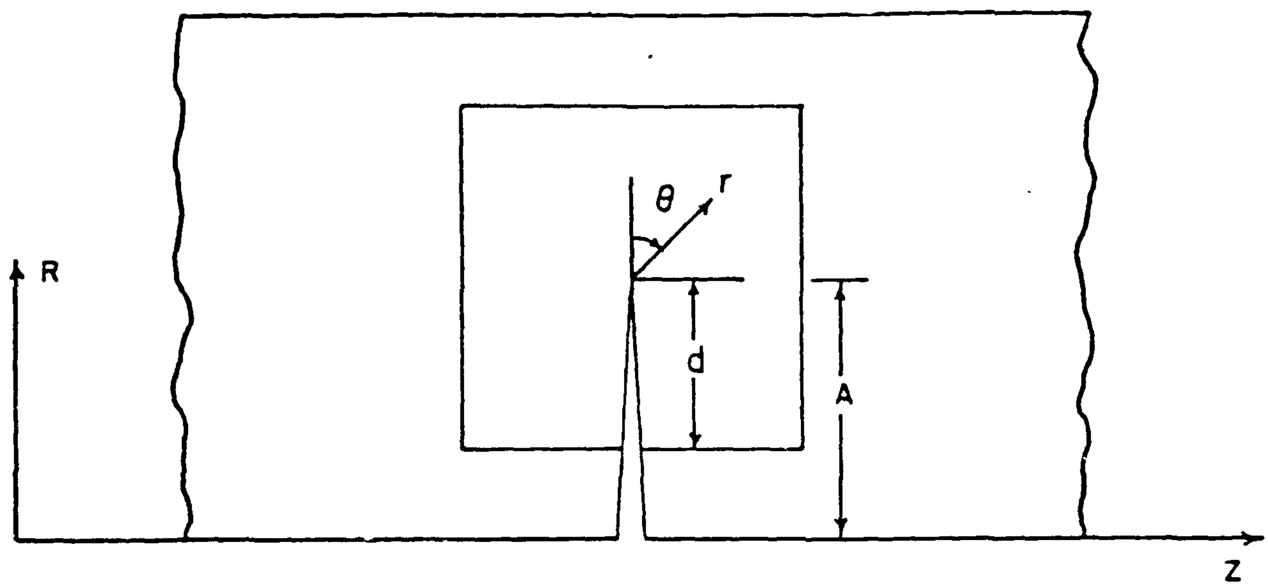


Figure 3. Crack-Tip Coordinates for Axisymmetric Models

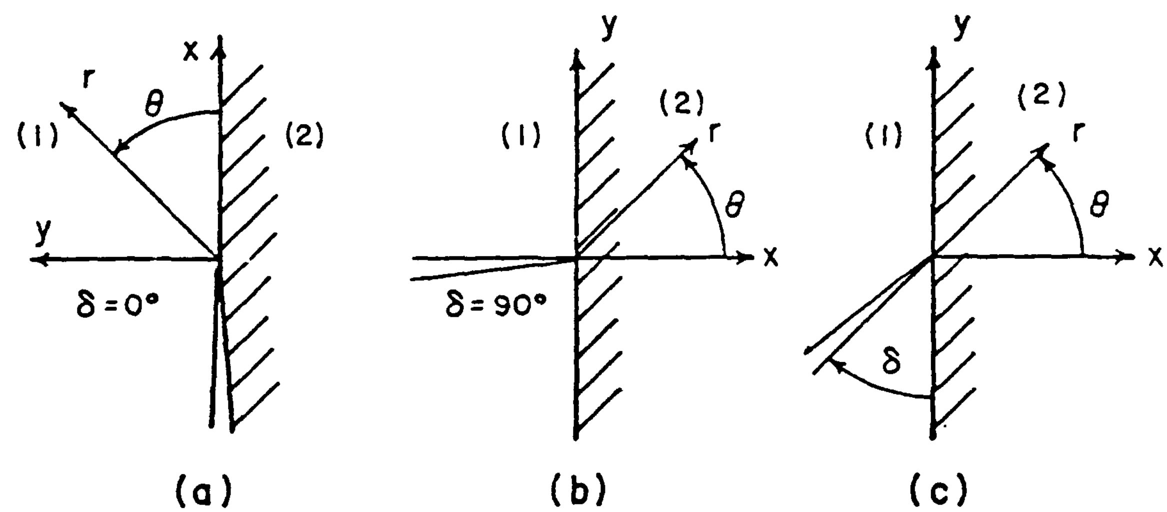


Figure 2. Cracks At the Bi-material Interface

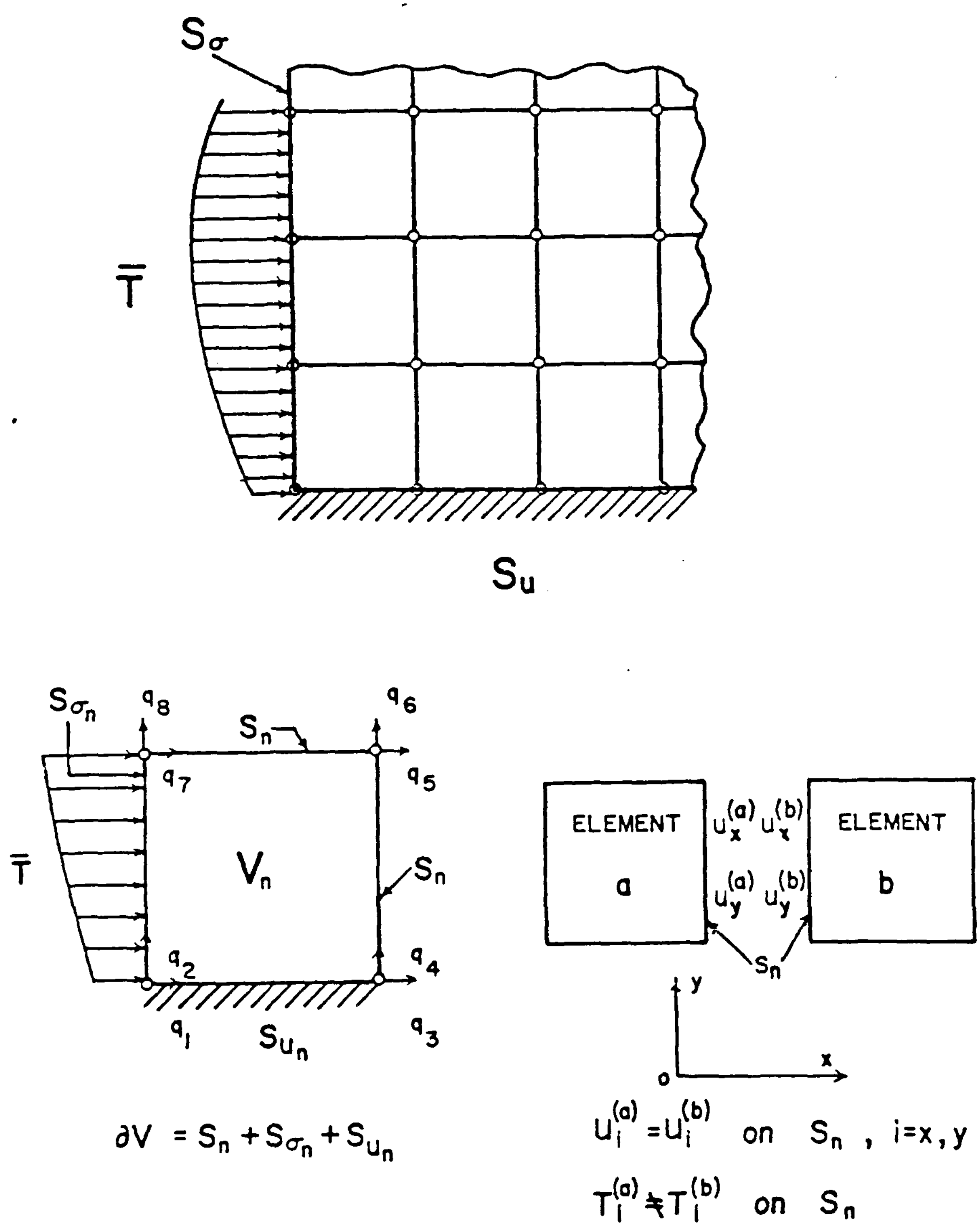


Figure 4. Notation for Hybrid Element Derivation

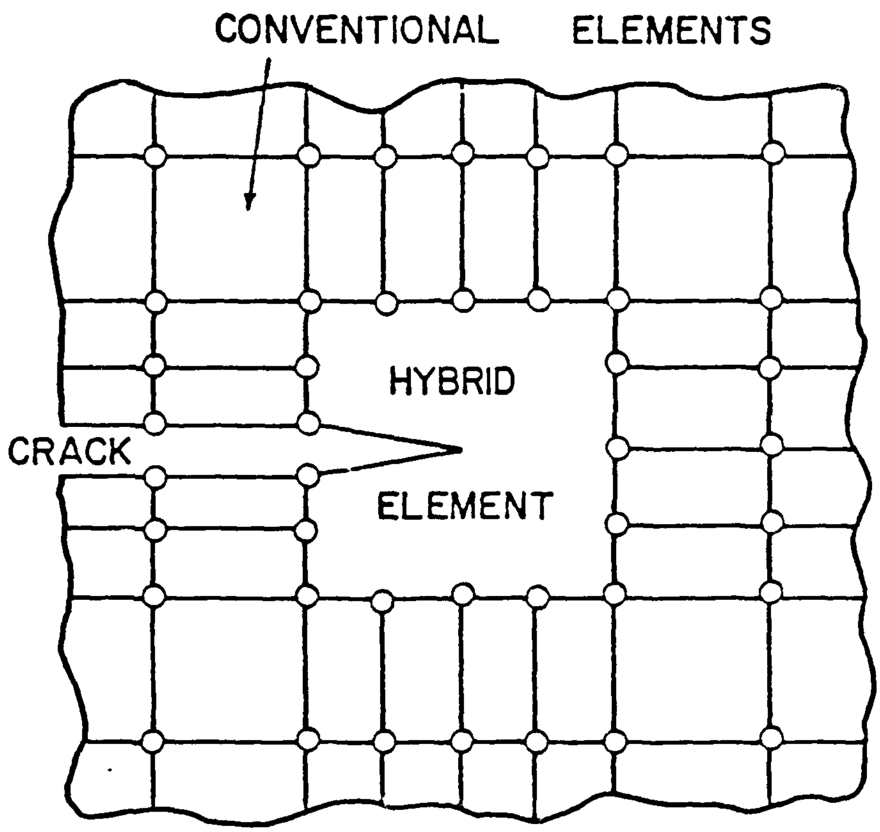
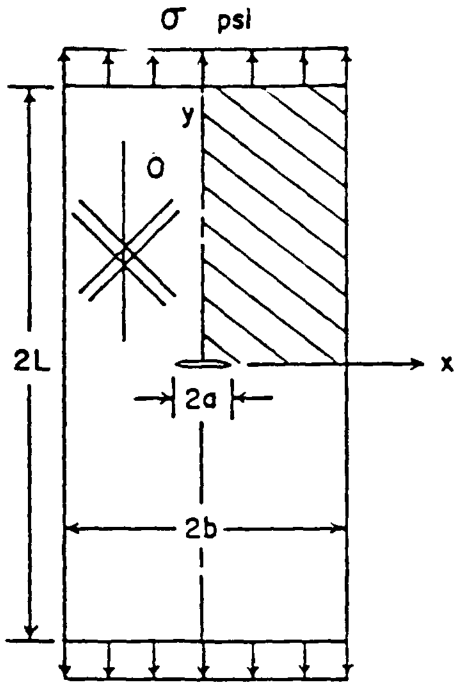
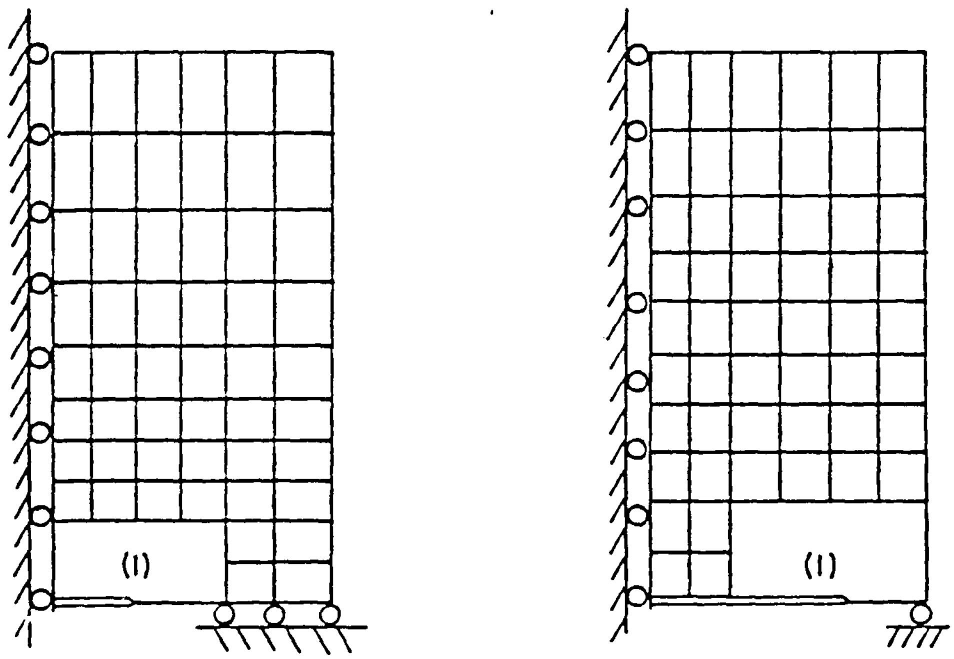


Figure 5. Combination of Hybrid and Conventional Elements to Model Cracked Structures



(a) Geometry



Element (1): 9-node Hybrid Crack Element

(b) Mesh Plan for $a/b \leq 6$ (c) Mesh Plan for $a/b > 6$

Figure 6. Center Crack in a Rectangular Anisotropic Sheet

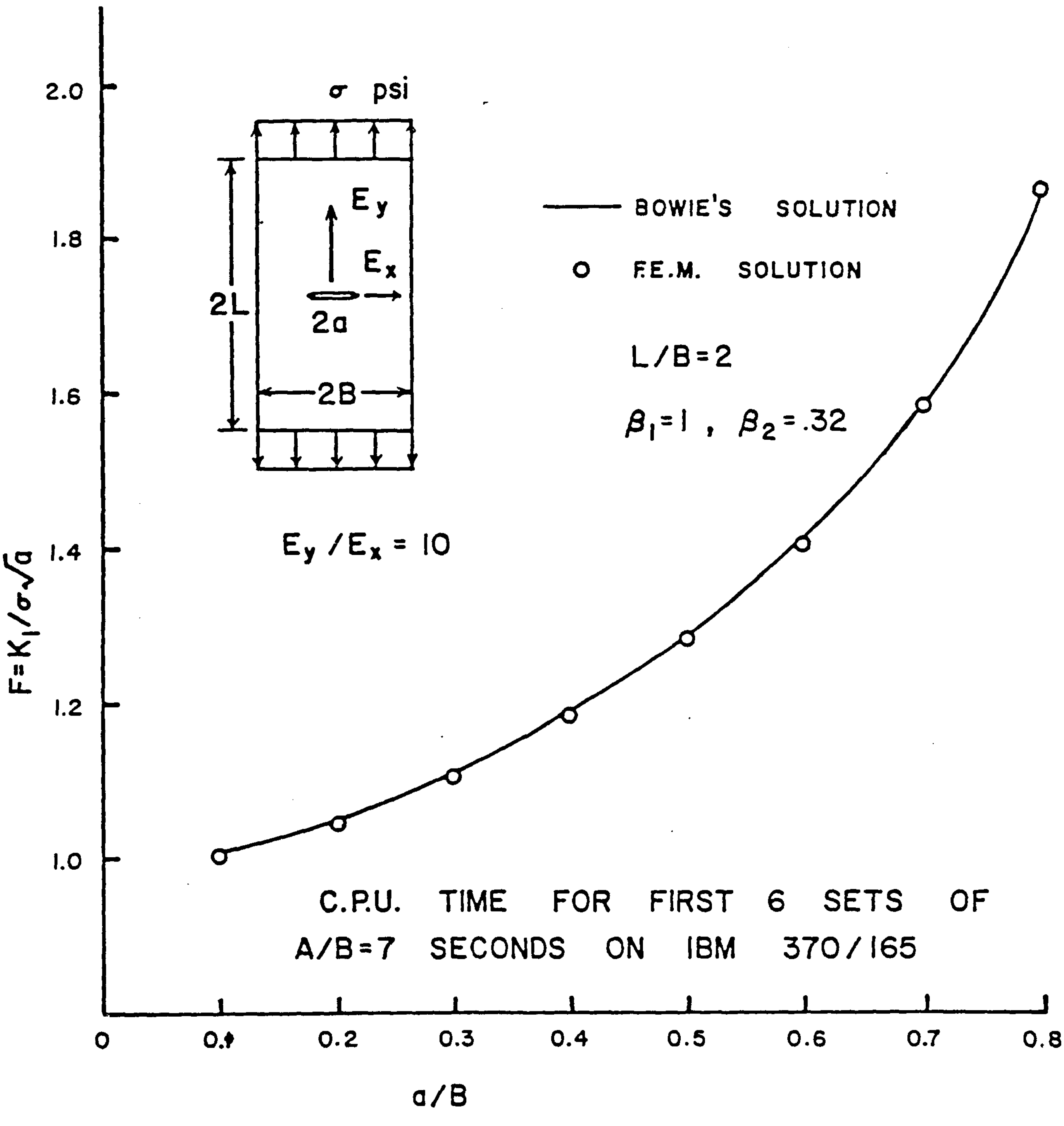


Figure 7. Accuracy Test of Present Finite Element Solutions

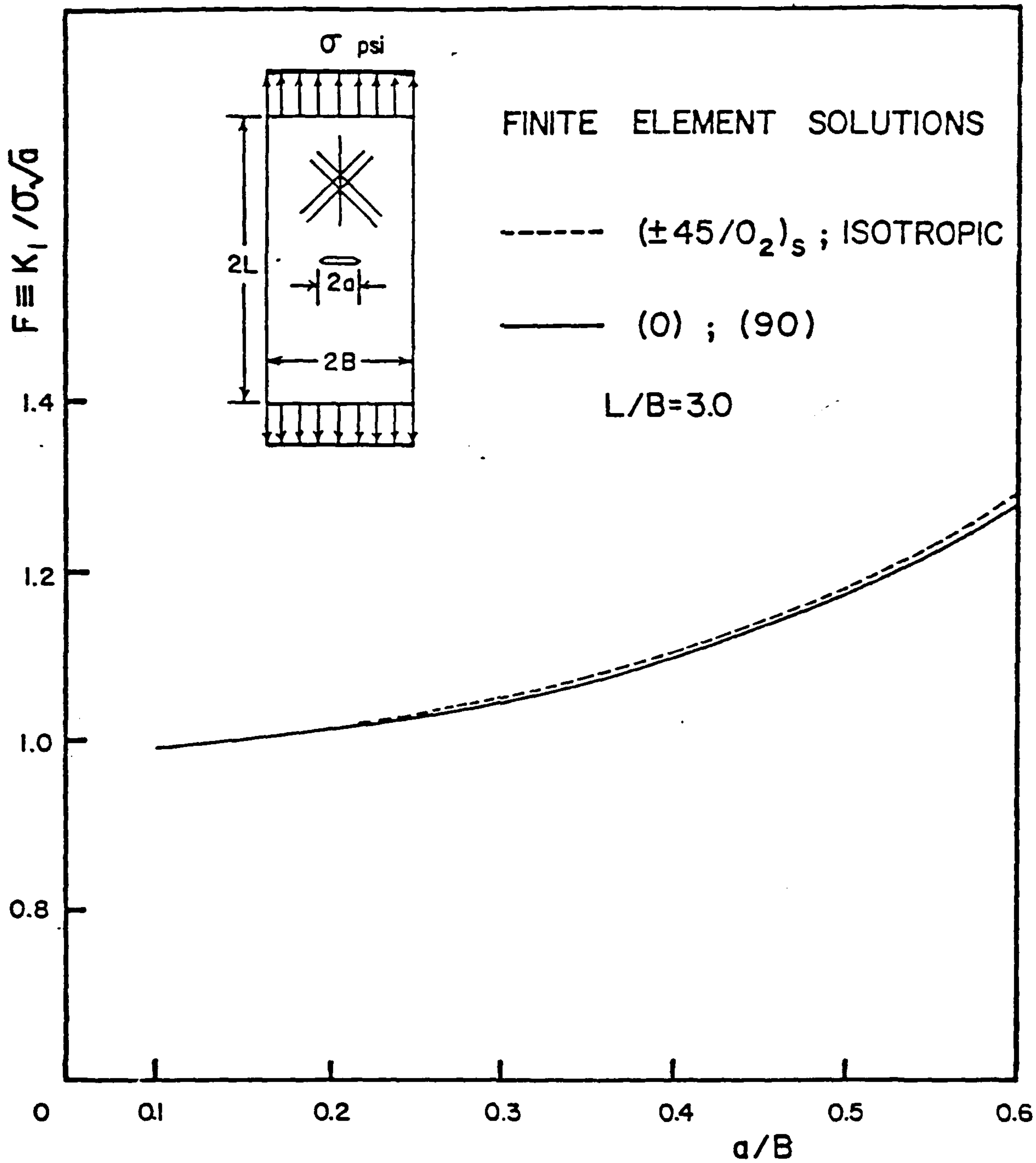


Figure 8. Stress Intensity Factors of a Center Crack with Boron/Aluminum Laminates Properties

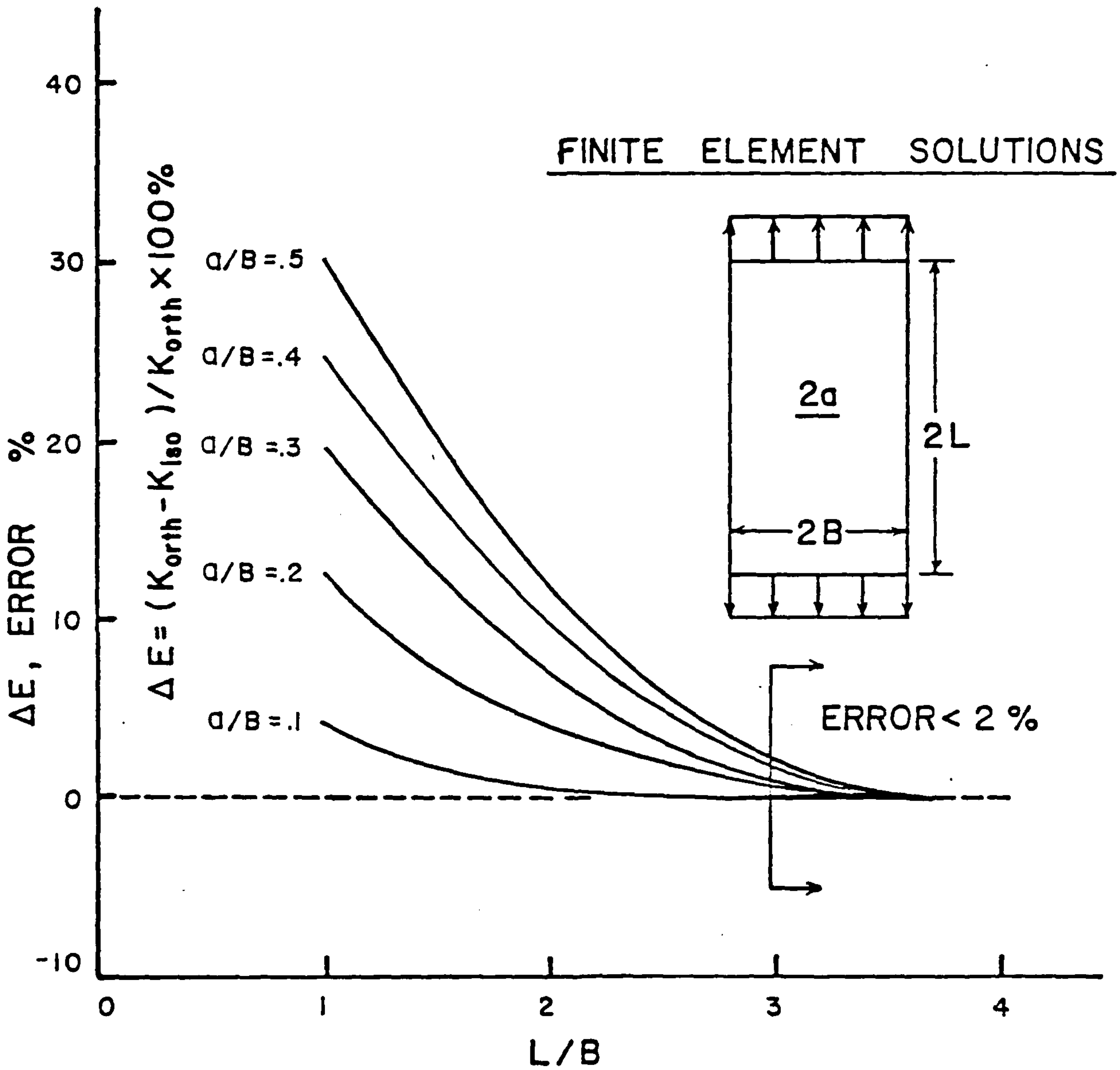


Figure 9. Errors in Approximating Isotropic Stress Intensity Factors for a Center Crack with Unidirectional Graphite/Epoxy Properties

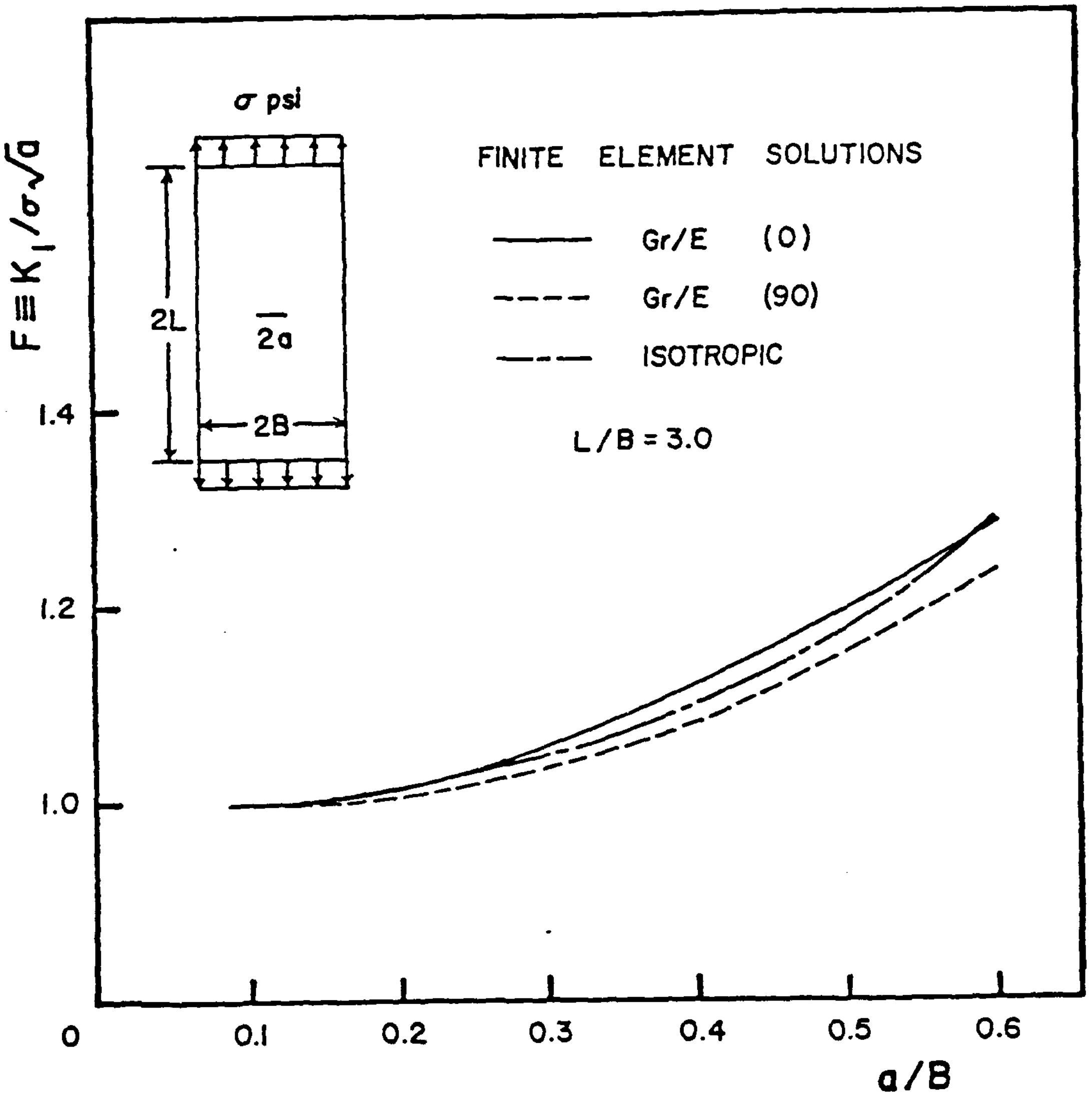


Figure 10. Stress Intensity Factors of a Center Crack with Graphite/Epoxy Properties

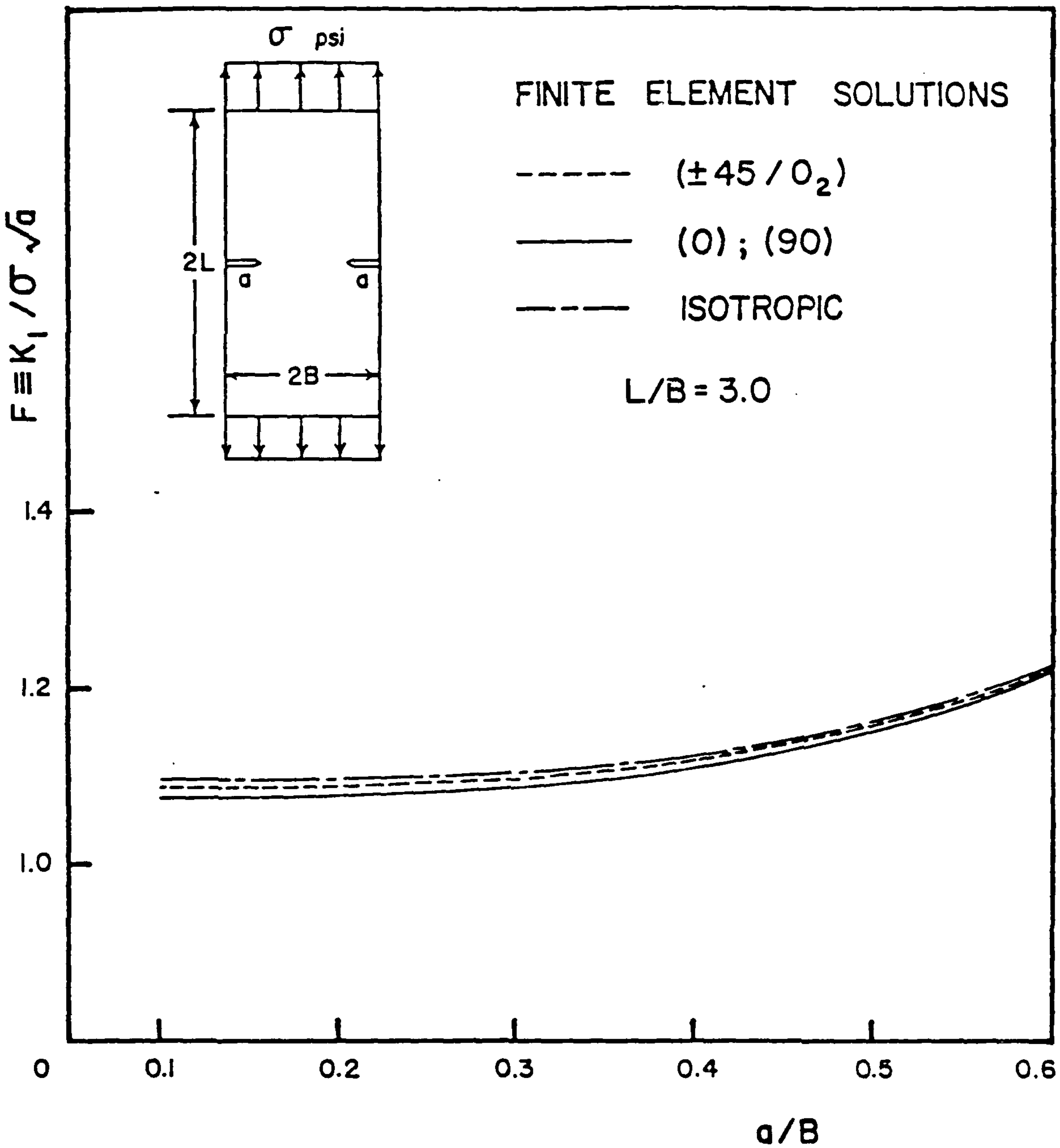


Figure 11. Stress Intensity Factors of a Double-Edged Crack with Boron/Aluminum Properties

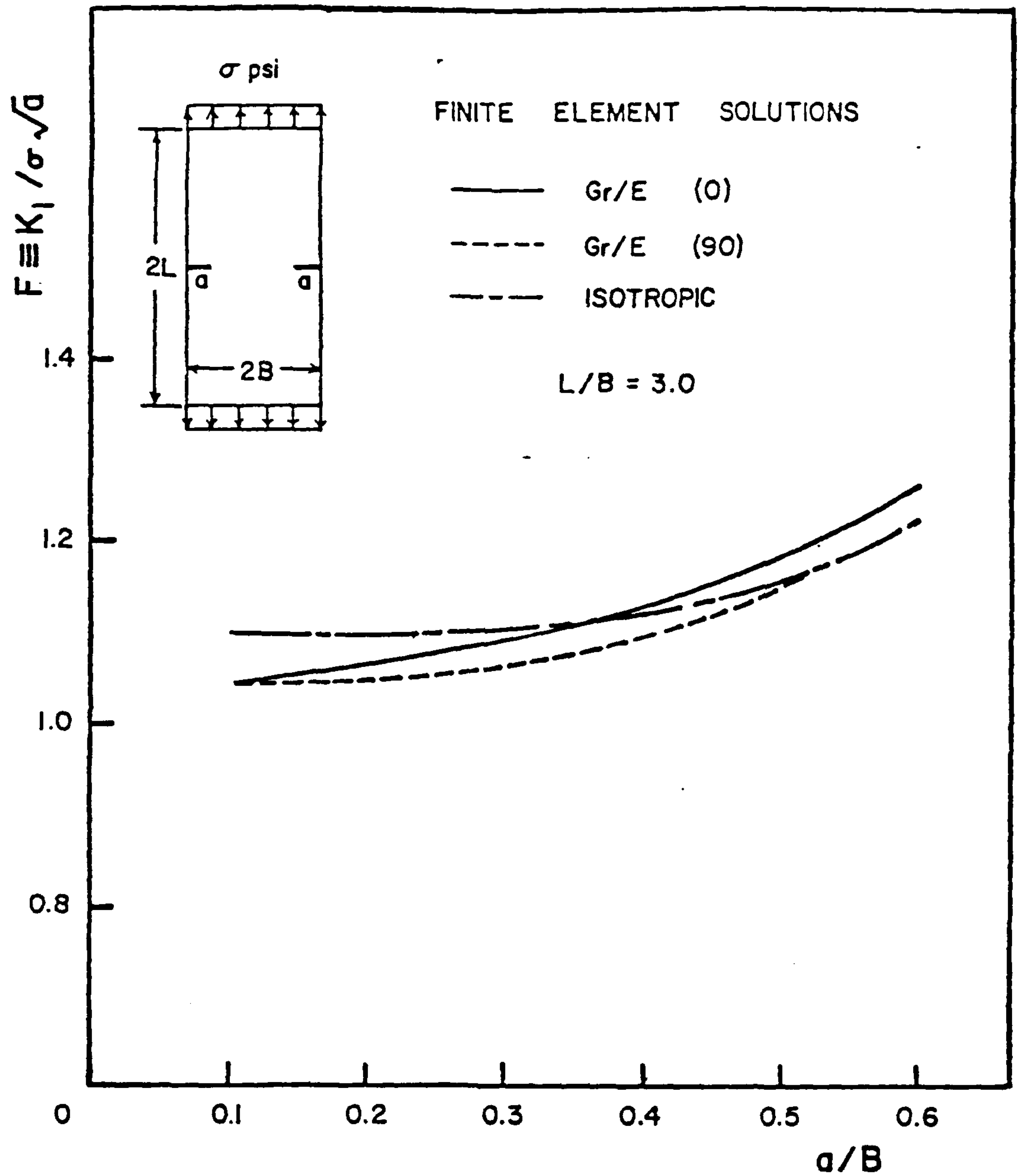


Figure 12. Stress Intensity Factors of a Double-Edge Crack with Graphite/Epoxy Properties

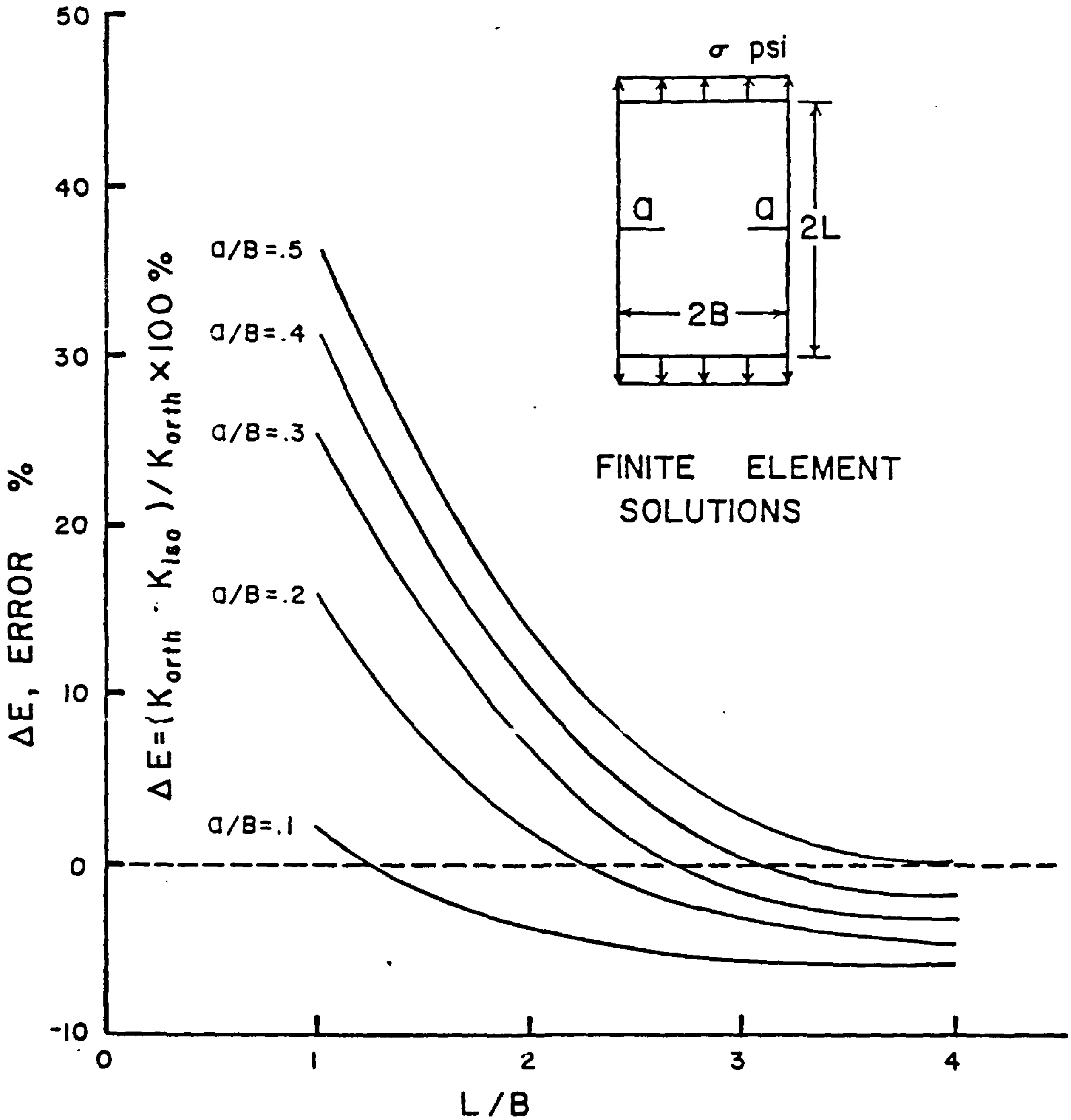
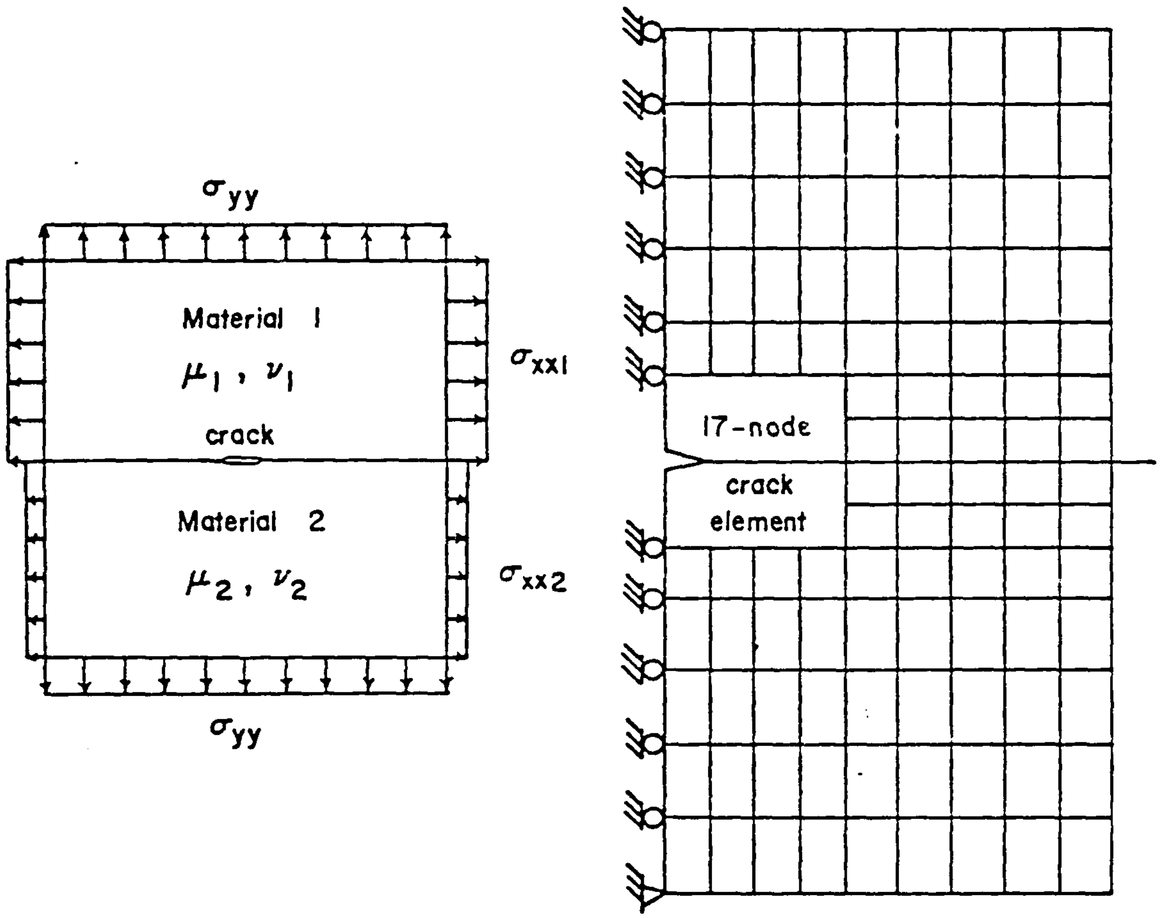


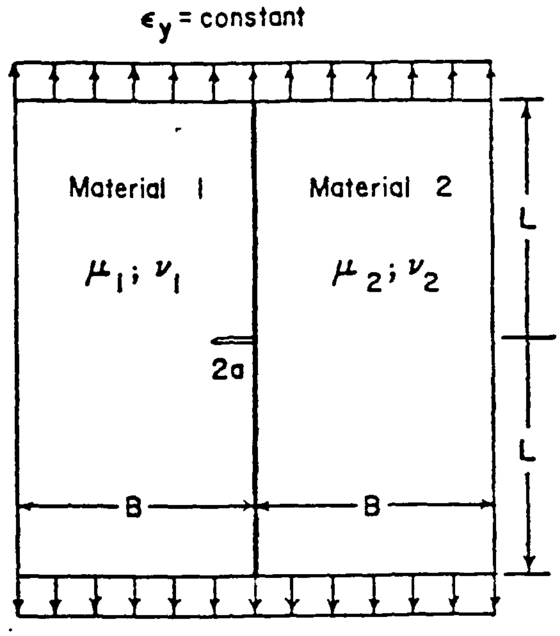
Figure 13. Errors in Approximating the Isotropic Stress Intensity Factors for a Double-Edge Crack With Unidirectional Graphite/Epoxy Properties



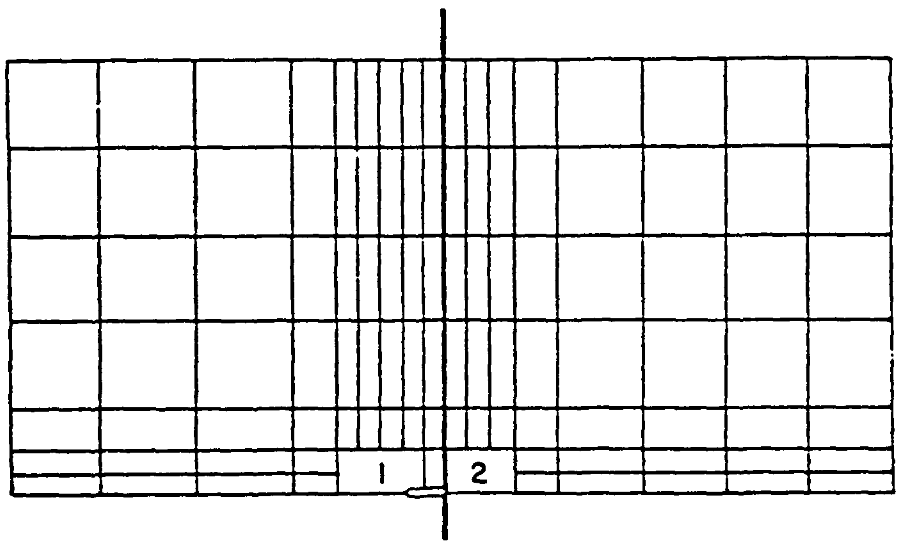
(a)
Geometry

(b)
Mesh Plan

Figure 14. A Debonding Crack Along the Bi-material Interface



(a) Geometry



(b) Mesh Plan

Figure 15. A Normal Crack Terminating at the Bi-material Interface

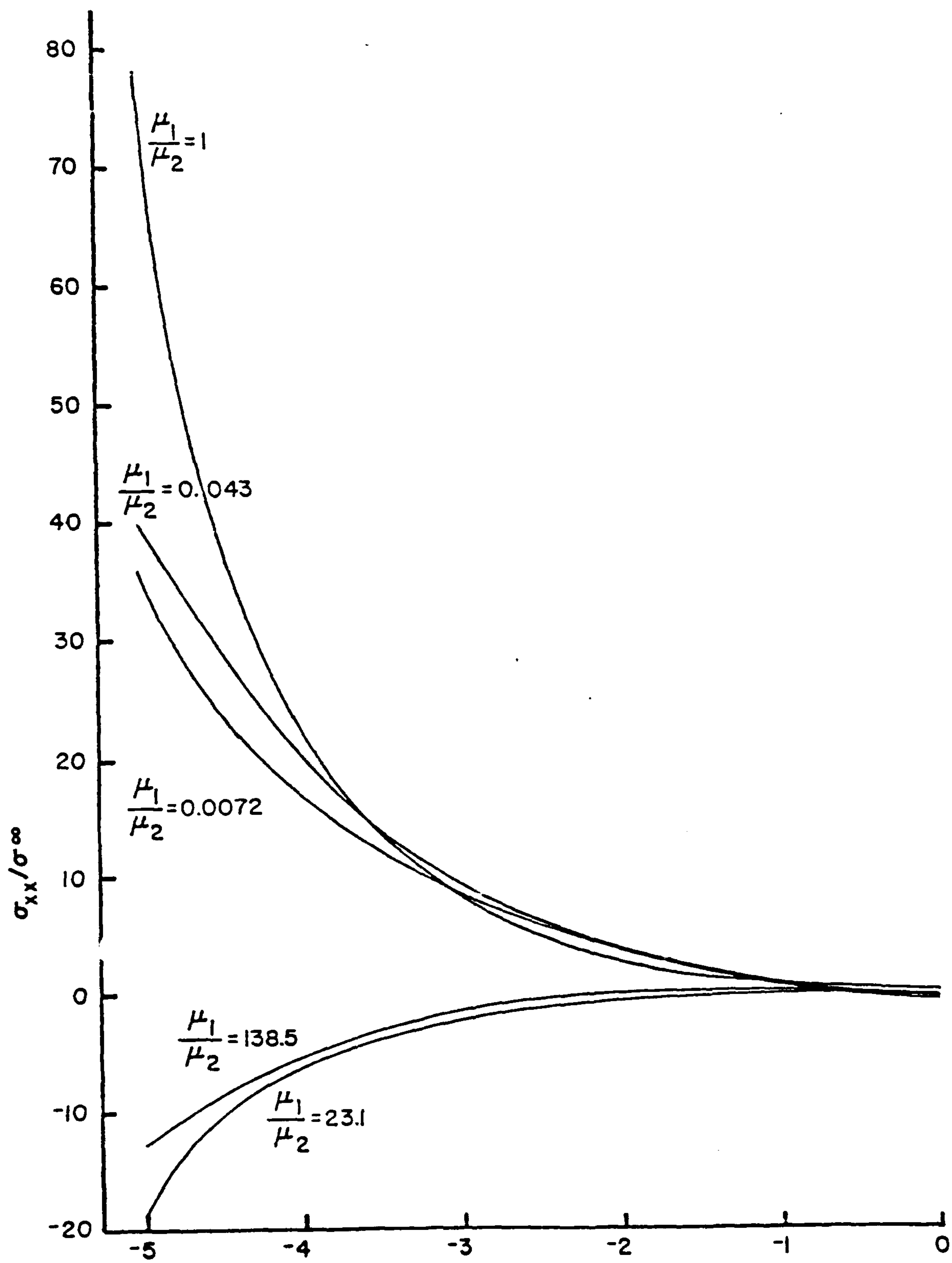
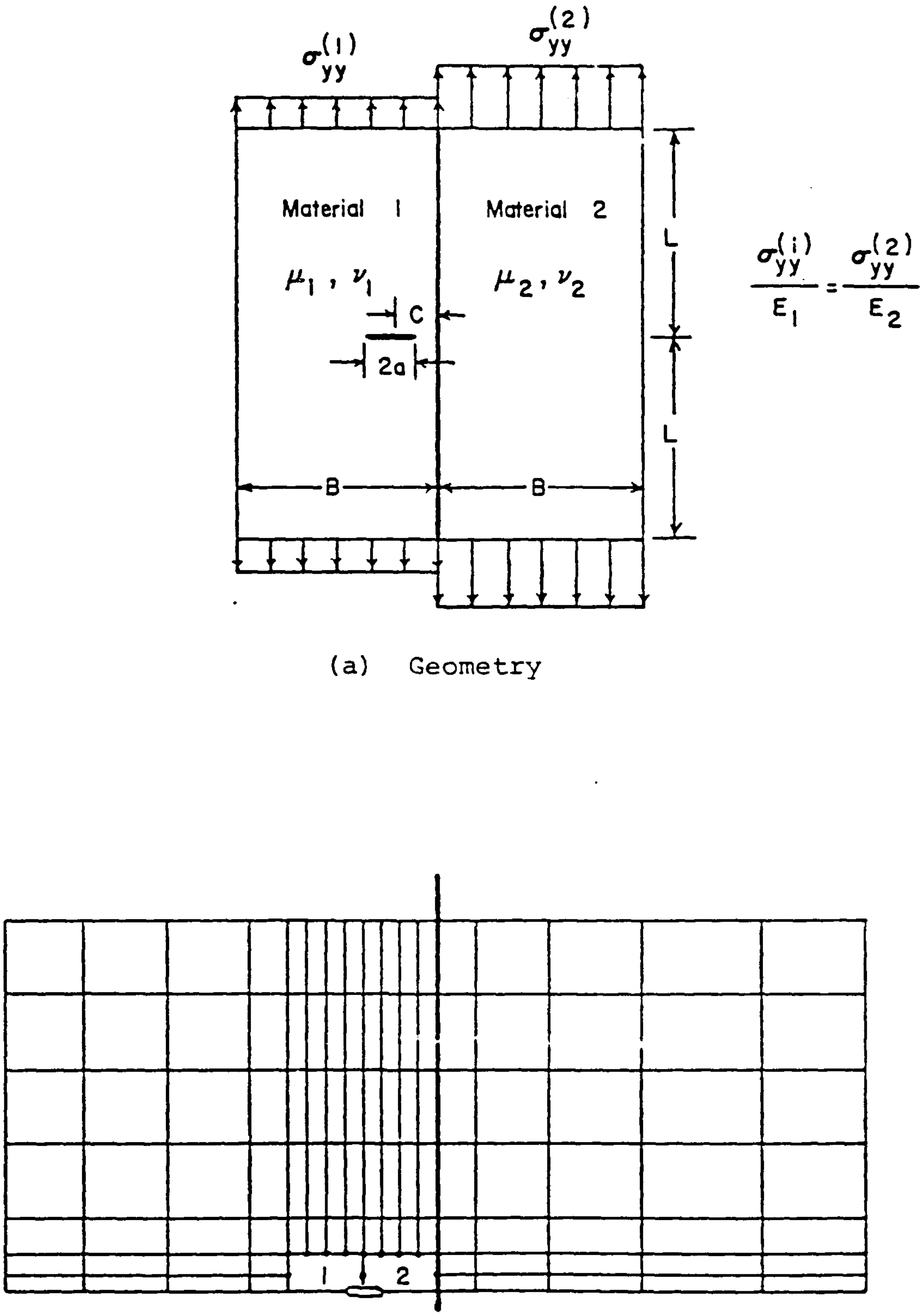


Figure 16. Transverse Stress σ_{xx} along the Bi-material Interface



(a) Geometry

(b) Mesh Plan

Figure 17. A Crack Near a Bi-material Interface

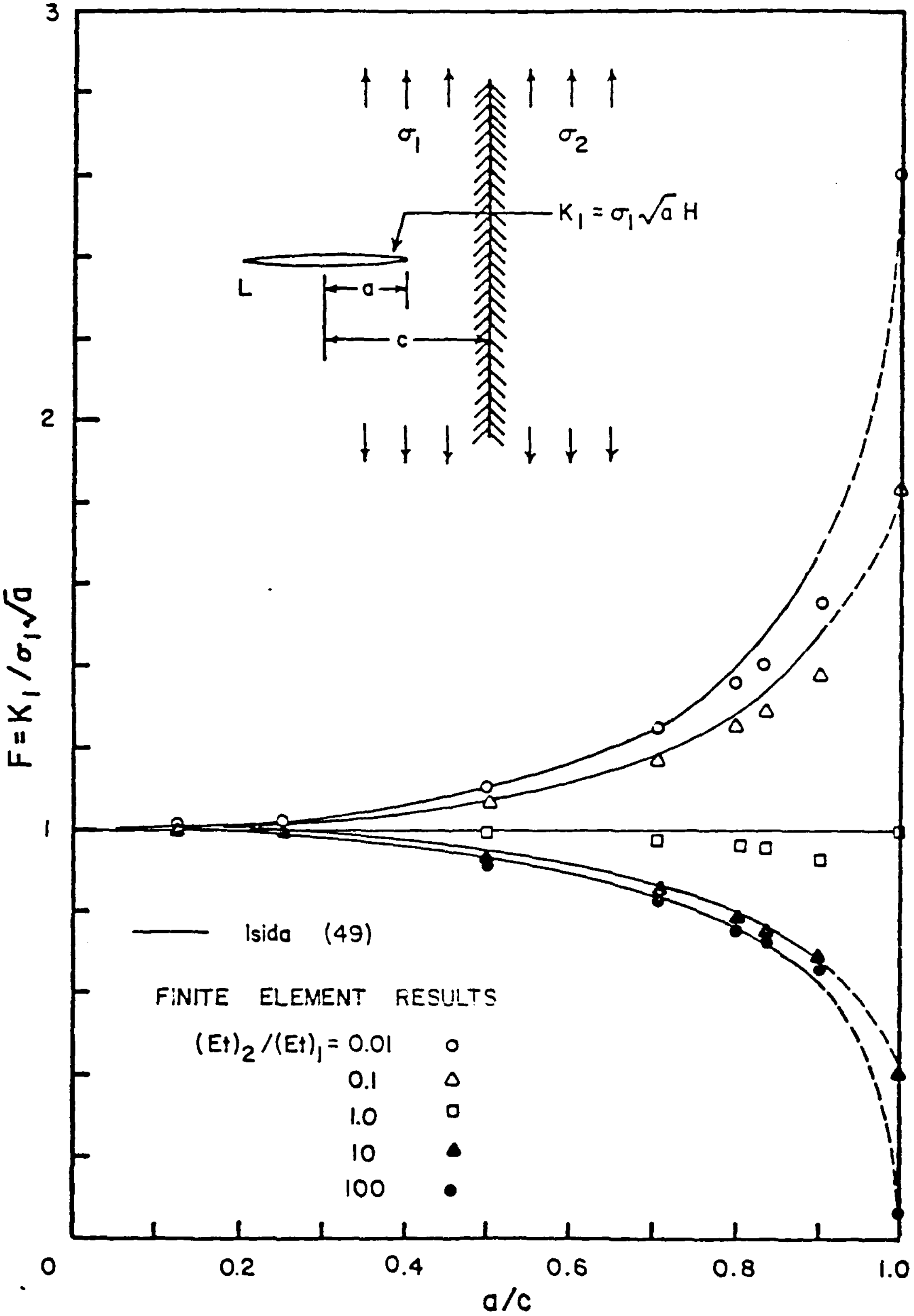
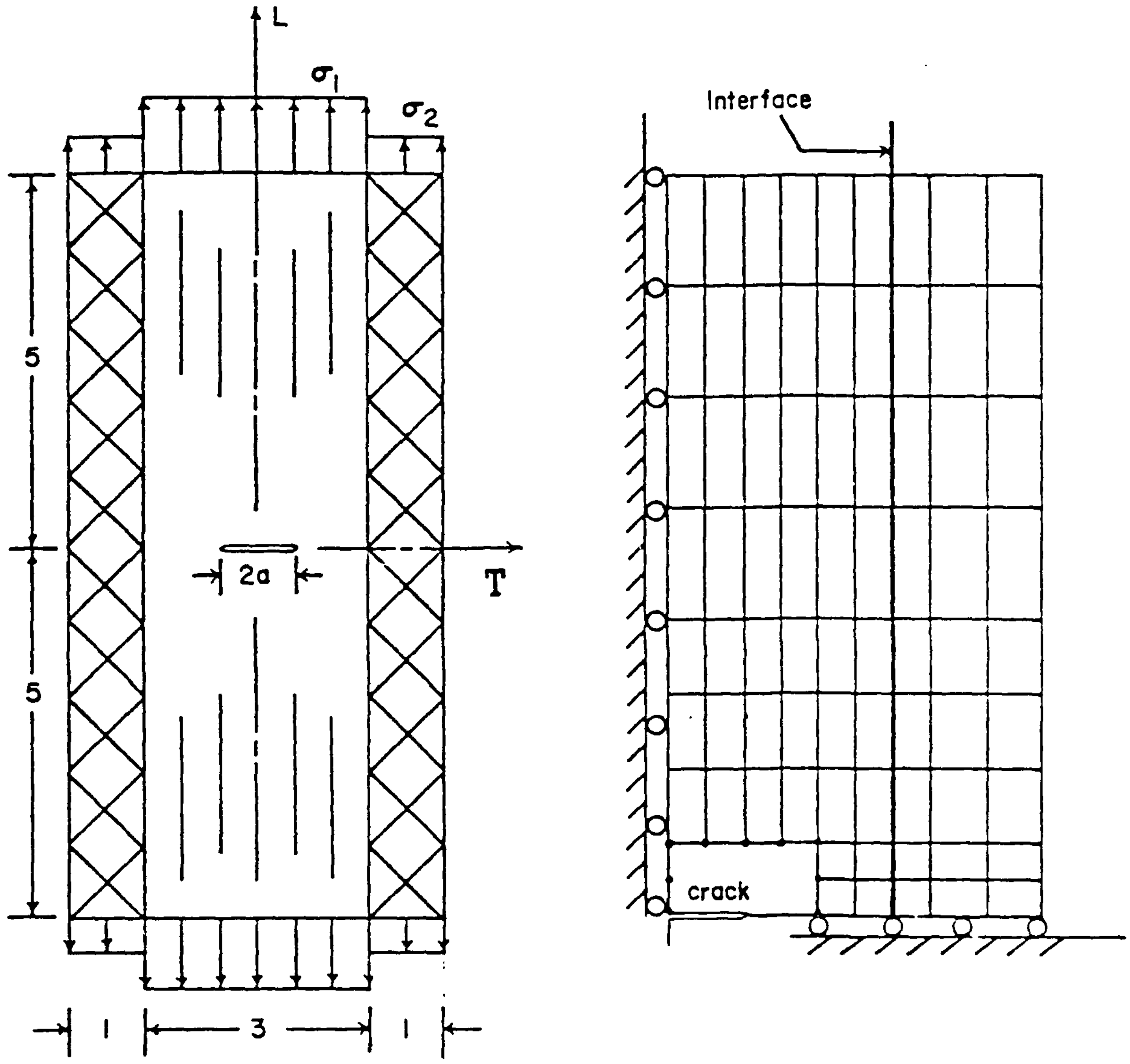


Figure 18. Comparison of Finite Element Solutions with Other Solutions



(a) Geometry

(b) Mesh Plan

Figure 19. A Crack in a Softener Strip

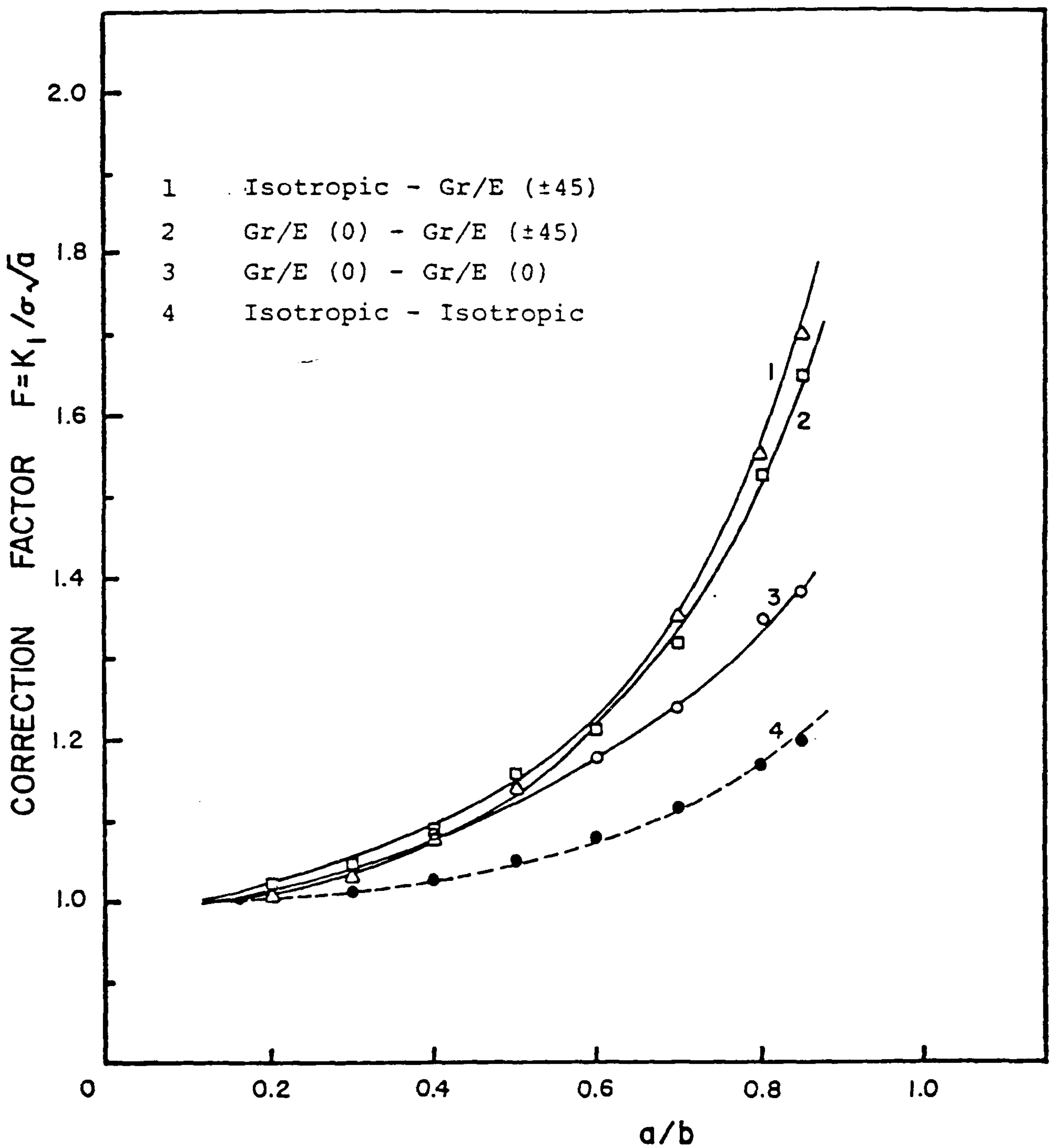
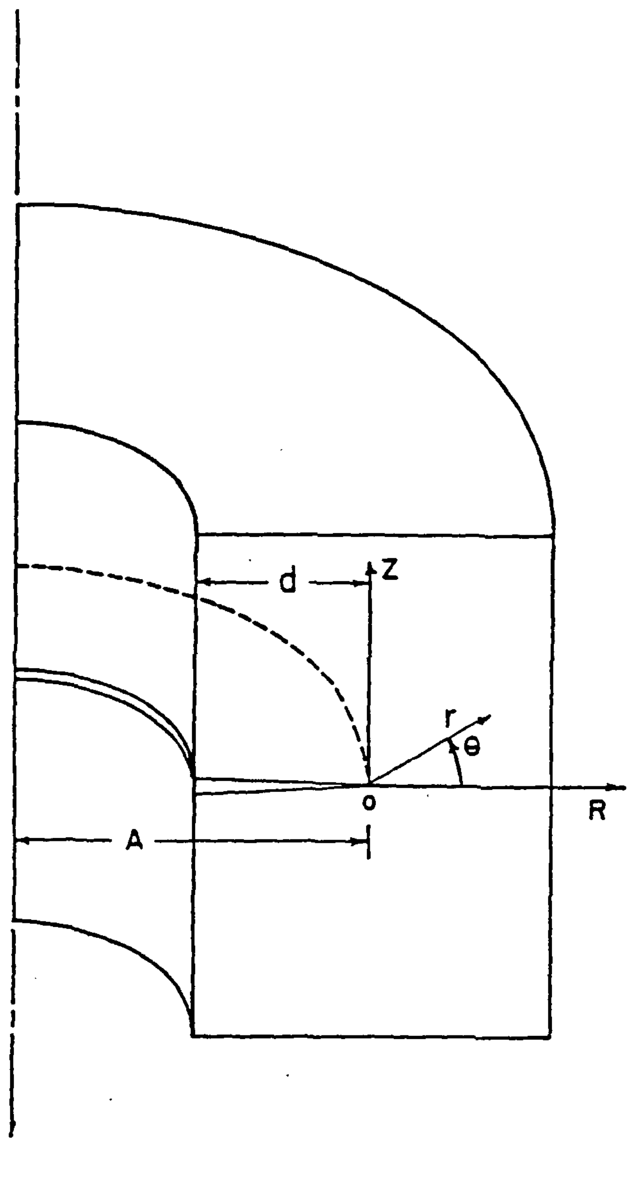


Figure 20. Finite Element Solutions of Stress Intensity Factors of a Center Crack in a Softener Strip



(a) Axisymmetric Crack Element

(b) Mesh Plane

Figure 21. Finite Element Model for Axisymmetric Crack Problems

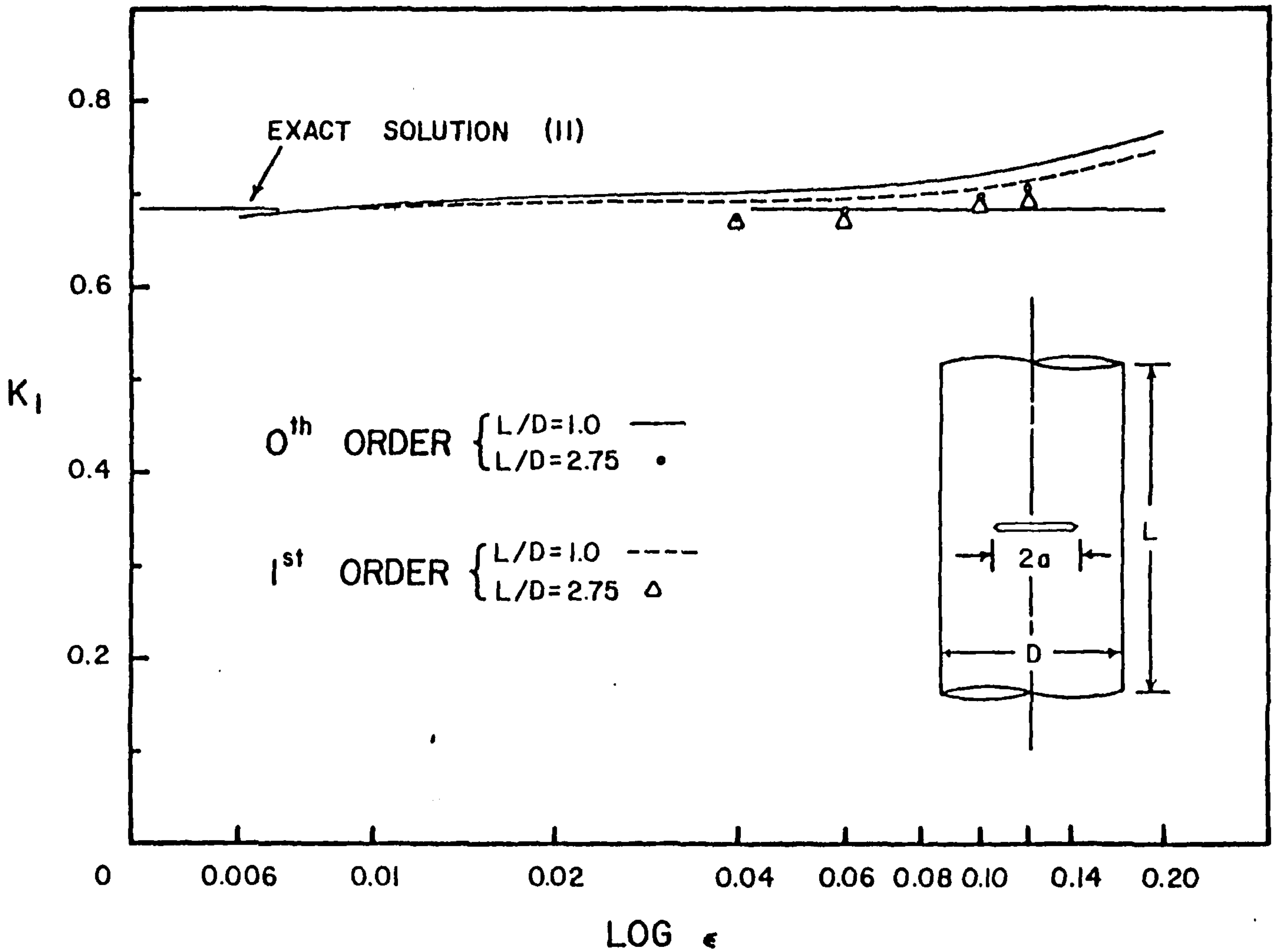


Figure 22. Finite Element Solutions of a Penny-Shaped Crack

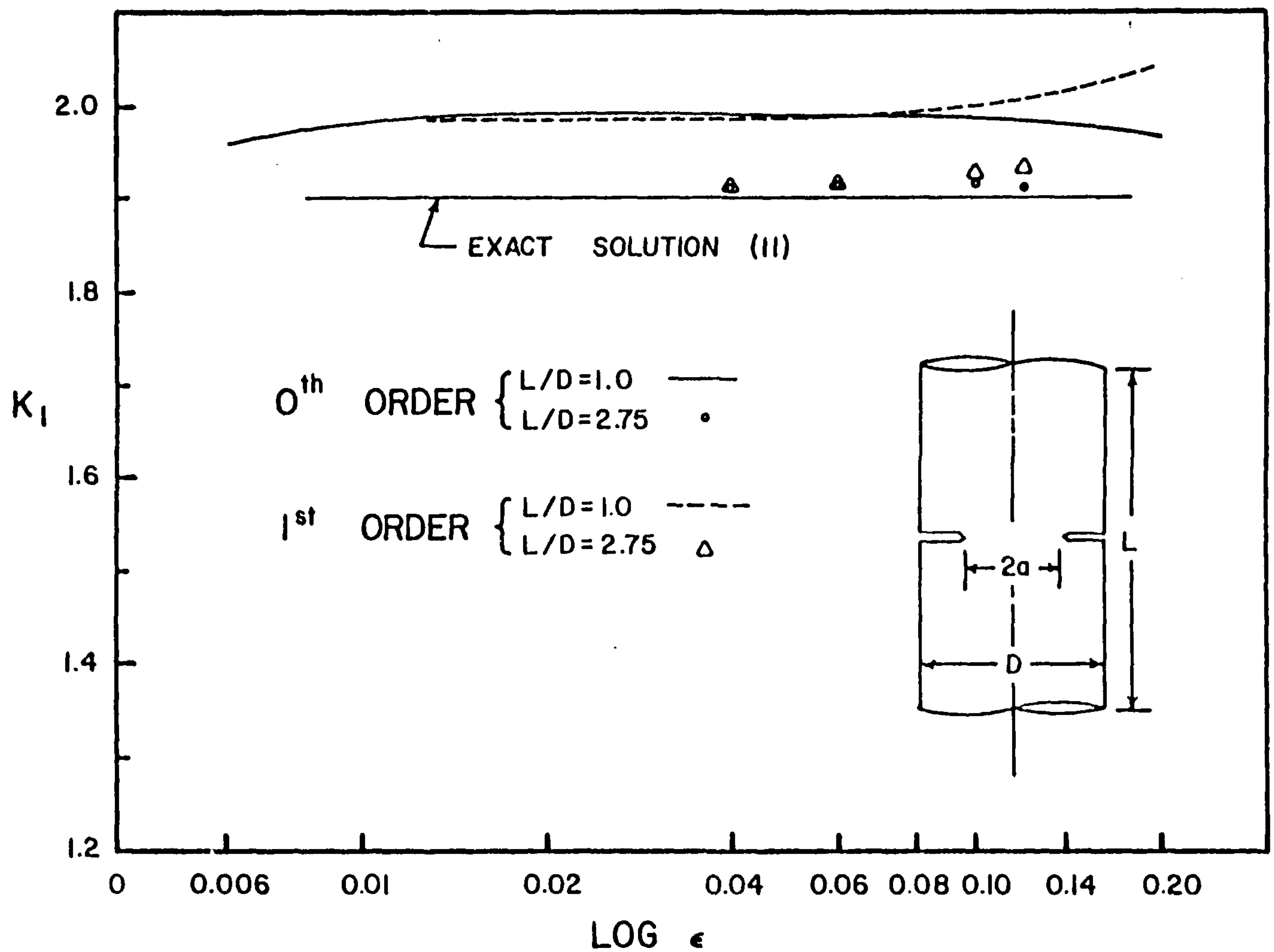


Figure 23. Finite Element Solutions of an Anisymmetric Circumferential Crack

Table 1. Roots λ of Characteristic equation (8)--to be Used for Finite Element Analysis

$\nu_1 = .35$, $\nu_2 = .30$ for $\mu_1/\mu_2 < 1$

$\nu_1 = .30$, $\nu_2 = .35$ for $\mu_1/\mu_2 > 1$, plane stress

μ_1/μ_2	λ_1	λ_2	λ_3	λ_4	λ_5
0	.73835	(1.7179,.4871)	(3.8098,1.0813)	(5.8499,1.3581)	(7.8742,1.5468)
.0072	.73345	(1.7205,.4850)	(3.8105,1.0779)	(5.8503,1.3546)	(7.8745,1.5433)
.0433	.71103	(1.7324,.4734)	(3.8140,1.0609)	(5.8524,1.3372)	(7.8760,1.5258)
.50	.56383	(1.8506,.1858)	(3.8631,0.7902)	(5.8831,1.0650)	(7.8985,1.2534)
1.00	.50000	(1.5000,0.0)	(2.5000,0.0)	(3.5000,0.0)	(4.5000,0.0)
2.00	.42944	(1.3178,0.0)	(2.8413,0.6502)	(4.8664,1.0067)	(6.8861,1.2283)
23.08	.17577	(1.0398,0.0)	(2.7490,1.0749)	(4.8136,1.4212)	(6.8489,1.6394)
138.46	.07491	(1.0069,0.0)	(2.7412,1.1115)	(4.8092,1.4567)	(6.8458,1.6745)
∞	0.0	(1.0000,0.0)	(2.7396,1.1190)	(4.8083,1.4639)	(6.8451,1.6816)

Table 2 Accuracy of Finite Element Solution

a/B	H*	
	F.E.M.	Bowie Solution [47]
.10	1.00	1.01
.20	1.04	1.05
.30	1.10	1.11
.40	1.18	1.19
.50	1.28	1.29
.60	1.40	1.41
.70	1.58	1.58
.80	1.86	1.85

* 1. $H = K_I/\sigma\sqrt{a}$

2. $\beta_1 = 1$, $\beta_2 = .32$, $L/B = 2.0$

Table 3. Stress Intensity Factors k_1 and k_2 for a Crack Along the Bi-Material Interface

$E_1 = 1$ psi, $\nu_1 = 0.30$, $\sigma_{yy} = 1$ psi, $\sigma_{xx1} = 1$ psi

20 x 20-inch panel, crack length = 2 in., plane stress

μ_1/μ_2	ν_2	σ_{xx_2} (psi)	k_1		k_2	
			EXACT*	F.E.M.**	EXACT*	F.E.M.
1	.3	1.00	1.000	1.01	0.0	0.0
3	.3	.53	.988	.997	.0724	.0728
10	.3	.37	.968	.975	.1171	.1176
23.1	.35	.38	.966	.972	.1208	.1213
100	.3	.31	.953	.959	.1391	.1395
144.2	.35	.36	.960	.965	.1303	.1306
1,000	.3	.30	.952	.957	.1415	.1418

*Solution for an infinite panel, from Rice and Sih [25].

**Finite element solution.

Table 4. ENERGY RELEASE RATE G FOR A CRACK ALONG THE BI-MATERIAL INTERFACE

$\mu_1/\mu_2 = 22.5$, $\nu_1 = 0.2$, $\nu_2 = 0.35$, $\sigma_{yy} = 1$ psi, $\sigma_{xx1} = 1$ psi

$\sigma_{xx2} = 0.39$ psi, 20 x 20-inch panel, plane stress

Half Crack Length (in.)	Half-panel Strain Energy	$G = dU/dc$		k_1		k_2	
		EXACT*	F.E.M.**	EXACT*	F.E.M.	EXACT*	F.E.M.
0.80	969.1628	25.04	---	0.862	0.867	0.125	0.122
0.81	969.4165	25.35	25.37	0.868	0.872	0.124	0.122
0.82	969.6734	25.67	25.69	0.873	0.878	0.124	0.122
0.84	970.1970	26.29	26.18	0.884	0.889	0.124	0.122
0.86	970.7337	26.92	26.84	0.895	0.900	0.123	0.121

*From Reference [48]

**$\Delta U/\Delta C$ calculated by taking differences between half crack length and half-panel strain energy, vising consecutive lines in the table.

Table 5. ENERGY RELEASE RATE G FOR HOMOGENEOUS MATERIAL

$E = 1$ psi, $\sigma_{yy} = \sigma_{xx} = 1$ psi, $\nu = 0.30$, 20 x 20-inch panel, plane stress

Half Crack Length (in.)	Half-panel Strain Energy	$G = dU/dc$		k_1	
		EXACT	F.E.M.	EXACT	F.E.M.
0.80	141.0115	2.51	---	.894	.900
0.81	141.0370	2.55	2.55	.900	.906
0.82	141.0629	2.58	2.59	.906	.911
0.84	141.1156	2.64	2.64	.917	.923
0.86	141.1697	2.70	2.71	.927	.934

Table 6. Mode 1 Stress Intensity Factors k_B and k_H for a Crack Normal to the Bi-Material Interface

E_1 = 1 psi, 40 x 40-inch panel, crack length (2a) = 2 in.

μ_1/μ_2	λ_1	Prescribed Stresses σ_{yy1}=1 psi, $\sigma_{yy2}=E_2$		Prescribed Displacement V_p = 20 in.		Reference [24]*	
		$k_B/(a)^{1-\lambda_1}$	$k_H/\sqrt{a}$	$k_B/(a)^{1-\lambda_1}$	$k_H/\sqrt{a}$	$k_B/(a)^{1-\lambda_1}$	$k_H/\sqrt{a}$
.0072	.7335	4.978	.833	4.978	.835	4.922	.871
.0433	.7110	4.241	.855	4.239	.854	4.176	.879
1	.500	.995	.995	.989	.989	1.00	1.00
23.08	.1758	.095	1.371	.091	1.318	.074	1.353
138.46	.0749	.0196	1.529	.0185	1.510	.0079	1.509

*Solution for an infinite plate with pressure loading (σ_{yy1} = 1 psi) over crack surfaces.

**ν_1 = .35, ν_2 = .30 for μ_1/μ_2 < 1

ν_1 = .30, ν_2 = .35 for μ_1/μ_2 > 1

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APPENDIX A.1

Stress distribution near the bi-material crack tip

I. Crack lying along the interface

For $\lambda_n = (\frac{2n-1}{2}) + i\lambda_j$, n = 1, 2, 3 ,-----

$$\sigma_x = \sum_n Re\left\{\lambda_n r^{\lambda_n-1} b_n\left[(\lambda_n+2)t e^{-i(\lambda_n-1)\theta} - \ell e^{i(\lambda_n-1)\theta} - (\lambda_n-1)t e^{-i(\lambda_n-3)\theta}\right]\right\}$$

$$\sigma_y = \sum_n Re\left\{\lambda_n r^{\lambda_n-1} b_n\left[-(\lambda_n-2)t e^{-i(\lambda_n-1)\theta} + \ell e^{i(\lambda_n-1)\theta} + (\lambda_n-1)t e^{-i(\lambda_n-3)\theta}\right]\right\}$$

$$\sigma_{xy} = \sum_n Re\left\{\lambda_n r^{\lambda_n-1} (i b_n)\left[-\lambda_n t e^{-i(\lambda_n-1)\theta} - \ell e^{i(\lambda_n-1)\theta} + (\lambda_n-1)t e^{-i(\lambda_n-3)\theta}\right]\right\}$$

For $\lambda_n = n$, n = 1, 2, 3 ,------

$$\sigma_x = \sum_n \lambda_n r^{\lambda_n-1} \varsigma\Big\{\left[(\lambda_n+3)\cos(\lambda_n-1)\theta - (\lambda_n-1)\cos(\lambda_n-3)\theta\right] C_n$$
$$+\left[(\lambda_n+1)\sin(\lambda_n-1)\theta - (\lambda_n-1)\sin(\lambda_n-3)\theta\right] d_n\Big\}$$

$$\sigma_y = \sum_n \lambda_n r^{\lambda_n-1} \varsigma\Big\{\left[-(\lambda_n-1)\cos(\lambda_n-1)\theta + (\lambda_n-1)\cos(\lambda_n-3)\theta\right] C_n$$
$$+\left[-(\lambda_n-3)\sin(\lambda_n-1)\theta + (\lambda_n-1)\sin(\lambda_n-3)\theta\right] d_n\Big\}$$

$$\sigma_{xy} = \sum_n \lambda_n r^{\lambda_n-1} \varsigma\Big\{\left[-(\lambda_n+1)\sin(\lambda_n-1)\theta + (\lambda_n-1)\sin(\lambda_n-3)\theta\right] C_n$$
$$+\left[(\lambda_n-1)\cos(\lambda_n-1)\theta - (\lambda_n-1)\cos(\lambda_n-3)\theta\right] d_n\Big\}$$

where

$\ell = 1$, $t = \frac{\beta-\alpha}{1+\alpha}$, $\varsigma = \beta$ in material 1

$\ell = \frac{\beta-\alpha}{1+\alpha}$, $t = 1$, $\varsigma = 1$ in material 2

II. Crack normal to the interface

$$\sigma_x = \sum_n \mathrm{Re}\left\{\lambda_n r^{\lambda_n - 1} b_n \left[(2f_R - g_R)\cos(\lambda_n - 1)\theta - (2f_I - g_I)\sin(\lambda_n - 1)\theta - (\lambda_n - 1)\left(f_R \cos(\lambda_n - 3)\theta - f_I \sin(\lambda_n - 3)\theta\right)\right]\right\}$$

$$\sigma_y = \sum_n \mathrm{Re}\left\{\lambda_n r^{\lambda_n - 1} b_n \left[(2f_R + g_R)\cos(\lambda_n - 1)\theta - (2f_I + g_I)\sin(\lambda_n - 1)\theta + (\lambda_n - 1)\left(f_R \cos(\lambda_n - 3)\theta - f_I \sin(\lambda_n - 3)\theta\right)\right]\right\}$$

$$\sigma_{xy} = \sum_n \mathrm{Re}\left\{\lambda_n r^{\lambda_n - 1} b_n \left[g_R \sin(\lambda_n - 1)\theta + g_I \cos(\lambda_n - 1)\theta + (\lambda_n - 1)\left(f_R \sin(\lambda_n - 3)\theta + f_I \cos(\lambda_n - 3)\theta\right)\right]\right\}$$

where

in material 1

$$f_R(\lambda_n) = \beta\left[1 + \alpha + (2\alpha\lambda_n - \alpha)\cos\lambda_n\pi\right] / D(\lambda_n)$$

$$f_I(\lambda_n) = -\beta\left[(2\alpha\lambda_n - \alpha)\sin\lambda_n\pi\right] / D(\lambda_n)$$

$$g_R(\lambda_n) = -\beta\left[\lambda_n + \alpha\lambda_n + (2\alpha\lambda_n^2 + \alpha\lambda_n - \alpha)\cos\lambda_n\pi + (1+\alpha)\cos 2\lambda_n\pi\right] /$$

$$g_I(\lambda_n) = \beta\left[(2\alpha\lambda_n^2 + \alpha\lambda_n - \alpha)\sin\lambda_n\pi + (1+\alpha)\sin 2\lambda_n\pi\right] / D(\lambda_n)$$

$$D(\lambda_n) = 1 + 2\alpha + 2\alpha^2 - 2(\alpha + \alpha^2)\cos\lambda_n\pi - 4\alpha^2\lambda_n^2$$

and in material 2

$$f_R = 1, \quad f_I = g_I = 0$$

$$g_R(\lambda_n) = \lambda_n - \cos\lambda_n\pi - \beta\left[\alpha + 2\lambda_n - (1 + 2\alpha - 4\alpha\lambda_n^2)\cos\lambda_n\pi + (1+\alpha)\cos 2\lambda_n\pi\right] /$$

APPENDIX A.2

Stresses and Displacements Used for the Derivation of Axisymmetric Crack Elements

$$\sigma_z = \sum_{j=1}^{N} \Big\{ \rho^{\lambda-2}\lambda(\lambda-1)\big[(3-\lambda-\gamma)\cos(\lambda-2)\theta + (\lambda-2)\cos(\lambda-4)\theta\big] + \frac{\varepsilon}{4}\rho^{\lambda-1}\lambda\big[\ell_1\cos(\lambda-1)\theta + (\lambda-1)(2-m)\cos(\lambda-3)\theta - (\lambda-1)(\lambda-2)\cos(\lambda-5)\theta\big]\Big\} b_j + O(\varepsilon^2)$$

$$\sigma_{rz} = \sum_{j=1}^{N} \Big\{ \rho^{\lambda-2}\lambda(\lambda-1)\big[(1-\lambda-\gamma)\sin(\lambda-2)\theta + (\lambda-2)\sin(\lambda-4)\theta\big] + \frac{\varepsilon}{4}\rho^{\lambda-1}\lambda\big[\ell_2\sin(\lambda-1)\theta + (\lambda-1)(2-m)\sin(\lambda-3)\theta - (\lambda-1)(\lambda-2)\sin(\lambda-5)\theta\big]\Big\} b_j + O(\varepsilon^2)$$

$$\sigma_r = \sum_{j=1}^{N} \Big\{ \rho^{\lambda-2}\lambda(\lambda-1)\big[(1+\lambda+\gamma)\cos(\lambda-2)\theta - (\lambda-2)\cos(\lambda-4)\theta\big] + \frac{\varepsilon}{4}\rho^{\lambda-1}\lambda\big[\ell_3\cos(\lambda-1)\theta - (\lambda-1)(2-m)\cos(\lambda-3)\theta + (\lambda-1)(\lambda-2)\cos(\lambda-5)\theta\big]\Big\} b_j + O(\varepsilon^2)$$

$$u_r = \sum_{j=1}^{N} \Big\{ -\frac{1}{2G}\rho^{\lambda-1}\lambda\big[-(\lambda+m-2+\gamma)\cos(\lambda-1)\theta + (\lambda-1)\cos(\lambda-3)\theta\big] + \frac{\rho^{\lambda}}{2G}\frac{\varepsilon}{4}\big[\ell_4\cos\lambda\theta + \lambda(\lambda-1)\cos(\lambda-4)\theta\big]\Big\} b_j + O(\varepsilon^2)$$

$$v_z = \sum_{j=1}^{N} \Big\{ \frac{1}{2G}\rho^{\lambda-1}\lambda\big[(-\lambda+m-\gamma)\sin(\lambda-1)\theta + (\lambda-1)\sin(\lambda-3)\theta\big] + \frac{\rho^{\lambda}}{2G}\frac{\varepsilon}{4}\big[\ell_5\sin\lambda\theta - 2\lambda(m-2)\sin(\lambda-2)\theta - \lambda(\lambda-1)\sin(\lambda-4)\theta\big]\Big\} b_j + O(\varepsilon^2)$$

where

(1) $\lambda = \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \ldots\ldots, t_1 = 2, \gamma = -1$

$$\ell_1 = (\lambda-4)^2 + m(\lambda-3) - \lambda$$

$$\ell_2 = (\lambda-2)^2 + m(\lambda-1) - \lambda$$

$$\ell_3 = (-\lambda^2+\lambda+16) - m(\lambda+5)$$

$$\ell_4 = 4m + 5\lambda - 4 - (m+\lambda)^2$$

$$\ell_5 = \lambda^2 - m^2 + 4m - 5\lambda$$

$$m = 4(1-\nu)$$

(2) $\lambda = 2, 3, 4, \ldots\ldots, t_1 = 0, \gamma = 1$

$$\ell_1 = (\lambda-2)^2 + m(\lambda-1) - \lambda$$

$$\ell_2 = m(\lambda+1) + \lambda(\lambda-1)$$

$$\ell_3 = (-\lambda^2 - 3\lambda + 12) - (\lambda+7)m$$

$$\ell_4 = \lambda - (m+\lambda)^2$$

$$\ell_5 = \lambda^2 - m^2 + 4m - 4 - \lambda$$

$$m = 4(1-\nu)$$

APPENDIX A.3

Relation Between the Stress Intensity Factors k_1, k_2 and the Constants b_j

$b_j = b'_j + ib'_{j+n}$, b'_j, b'_{j+n} are real constants. n = No. of stress (or displacement) terms assumed.

1. Homogeneous Isotropic Materials

$$k_1 = \sqrt{2}\, b'_1$$
$$k_2 = \sqrt{2}\, b'_{n+1}$$

2. Homogeneous Anisotropic Materials

$$k_1 = \sqrt{2}\,(s_2 - s_1)/s_2\; b'_1$$
$$k_2 = \sqrt{2}\,(s_2 - s_1)\, b'_{n+1}$$

3. A Debond Crack Along the Bi-Material Interface

$$k_1 = 2\sqrt{(\beta-\alpha)/(1+\alpha)}\,(.50\, b'_1 + \lambda_{1j}\, b'_{n+1})$$
$$k_2 = 2\sqrt{(\beta-\alpha)/(1+\alpha)}\,(-.50\, b'_{n+1} + \lambda_{1j}\, b'_1)$$

4. A Crack Normal to the Bi-Material Interface

$$k_1 = \sqrt{2}\,(1 + \lambda_1 + g_R(\lambda_1))\,\lambda_1\, b'_1$$

λ_1, $g_R(\lambda_1)$ is defined in Appendix 2.A.

5. Penny-Shaped or Circumferential Crack

$$k_1 = \frac{3}{\sqrt{2}}\, b'_1$$

APPENDIX A.4

Material Properties for the Finite Element Analysis

I. Boron/Aluminum

(i) (0) Layup:

$E_x = 20.1$ msi, $E_y = 33.1$ msi, $\nu_{yx} = .25$

$G_{xy} = 7$ msi, $S_1 = 1.5180i$, $S_2 = .5134i$

(ii) ($\pm 45/0_2$) Layup:

$E_x = 20.3$ msi, $E_y = 26.5$ msi, $\nu_{yx} = .33$

$G_{xy} = 8.7$ msi, $S_1 = 1.0848i$, $S_2 = .8068i$

(iii) (90) Layup:

$E_x = 33.1$ msi, $E_y = 20.1$ msi, $\nu_{yx} = .15$

$G_{xy} = 7$ msi, $S_1 = 1.9480i$, $S_2 = .6588i$

II. Graphite/Epoxy

(i) (0) Layup:

$E_x = 1.7$ msi, $E_y = 17$ msi, $\nu_{yx} = .21$

$G_{xy} = 0.65$ msi, $S_1 = 1.5918i$, $S_2 = .19866i$

(ii) (90) Layup:

$E_x = 17$ msi, $E_y = 1.7$ msi, $\nu_{yx} = .021$

$G_{xy} = .65$ msi, $S_1 = 5.0338i$, $S_2 = .6282i$

Appendix B

COUPLING ACTIVITIES

The Principal Investigator, Prof. James W. Mar, was very active during the years of this contract in the USAF Scientific Advisory Board (SAB). As a member of the SAB he was assigned at various times to the Division Advisory Groups of the Aeronautical Systems Division, the Armament Development and Test Center, the Foreign Technology Division, the Logistics Cross Panel, and the Science and Technology Advisory Group. Some of the advisory activities which were impacted by the research activities of this contract are as follows:

1. Chairman of the USAF SAB Ad Hoc Committee on Advanced Composites Technology, 1976.

2. Chairman of the USAF SAB Ad Hoc Committee on the B-1 Strucutre, 1973-1977.

3. Chairman of the USAF AFSC STAG Ad Hoc Committee on 3-D Carbon/Carbon Nozzles, 1977.

The Principal Investigator during this period was also a member of the following advisory Committees:

1. NASA Research and Technology Advisory Committee on Materials and Structures.

2. NASA Space Systems Committee.

3. DOD Defense Science Board Task Force on V/STOL.

4. AGARD Structures and Materials Panel.

Of special note, three students who participated in the research activities of this contract are now working in the field of advanced composites. Their present assignments are due in large measure to their

involvement with their research. They are listed as follows:

Dr. K. Y. Lin, Boeing Commercial Airplane Co.

Mr. David Maass, Sikorsky Aircraft

2nd Lt. Karyn Knoll, AFML